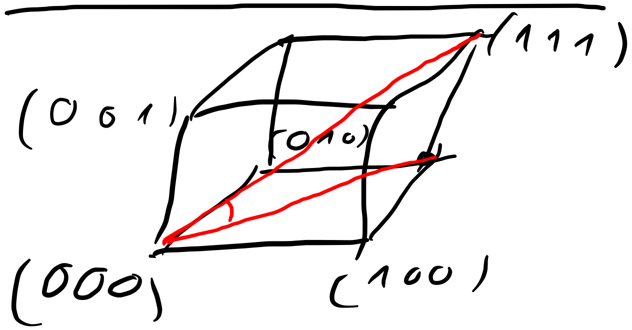
$$\langle b_i, c_j \rangle = \begin{cases} 1 & \text{ha } i = j \\ 0 & \text{ha } i \neq j \end{cases}$$

$$\begin{array}{cc} & B \\ & \begin{bmatrix} c_1 & \dots & c_n \end{bmatrix} \\ \underbrace{\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}}_{A} & \underbrace{\begin{bmatrix} \langle b_i, c_j \rangle \end{bmatrix}}_{E} = AB = E \end{array}$$

$$\underline{B = A^{-1}.}$$



$$\langle \underbrace{(1,1,\dots,1)}_{\sqrt{n}}, \underbrace{(1,0,\dots,0)}_{1} \rangle = 1 \quad \nearrow \text{él}$$

$$\underbrace{1/\sqrt{n}}_{\to 0} = \cos \alpha_n \qquad n \to \infty$$

$$\alpha_n \to 90^\circ$$

$z = 0$ lap

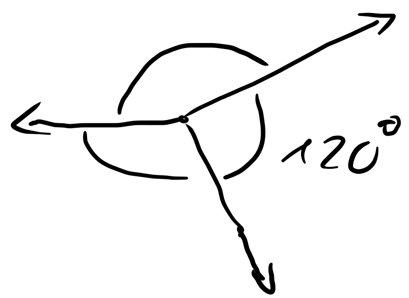
$(x_1, \dots, x_{n-1}, 0)$ lap $\quad 0 \le x_i \le 1$

$$\langle \underbrace{(1 \dots 1)}_{\sqrt{n}}, \underbrace{(1 \dots 1, 0)}_{\sqrt{n-1}} \rangle = n-1 \qquad \cos \beta_n = \frac{n-1}{\sqrt{n}\sqrt{n-1}} = \frac{\sqrt{n-1}}{\sqrt{n}} \to 1$$

$$\beta_n \to 0^\circ.$$

Freud 8. 2. 17

$60°$ n lehet tiss non,
$120°$ ≤ 3 lehet $\forall$ dimenzióban.



a, b, c itt egységvektorok
=> egy síkba esnek
$\|a+b+c\| = 0.$

$60°$ (F.)

$$AA^T = \begin{bmatrix} 1 & \cdots & 1/2 \\ & \ddots & \\ 1/2 & \cdots & 1 \end{bmatrix} \leftarrow \det \neq 0.$$

$$-3\bar{x}x + 4i\bar{y}x - 4i\bar{x}y + 3\bar{y}y - \bar{z}z = Q(x,y,z)$$

$$[\bar{x}\ \bar{y}]\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [\bar{x}a + \bar{y}c,\ \bar{x}b + \bar{y}d]\begin{bmatrix} x \\ y \end{bmatrix} =$$

$$= a\bar{x}x + b\bar{x}y + c\bar{y}x + d\bar{y}y$$

$$\begin{array}{c|cc} & x & y \\ \hline \bar{x} & a & b \\ \bar{y} & c & d \end{array}$$

$$\begin{bmatrix} -3 & -4i & 0 \\ 4i & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

önadjungált

értékkészlet valós.

sé $\quad 5, \; -5, \; 1$ $\qquad$ indefinit

$$\frac{1}{\sqrt5}\begin{bmatrix} 1 \\ 2i \end{bmatrix}, \; \frac{1}{\sqrt5}\begin{bmatrix} 2i \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$+5\left|\frac{x-2iy}{\sqrt5}\right|^2 - 5\left|\frac{-2ix+y}{\sqrt5}\right|^2 - 1\,\underset{|z|^2}{\underset{\shortparallel}{\bar z z}}$$

VII/6. $\sim$ mátrix

$$M = M_1 + i M_2 \qquad M_1, M_2 \text{ önadj.}$$

$$M^* = M_1^* + (i M_2)^* = M_1 + (-i) M_2$$

$$M_1 = \frac{1}{2}(M + M^*) \qquad M_2 = \frac{1}{2i}(M - M^*)$$

Q-ortog $e_1, \dots, e_n$ bázis Q ~ B szimm bilin

$B(x,x) = Q(x)$

M szimm mátrix

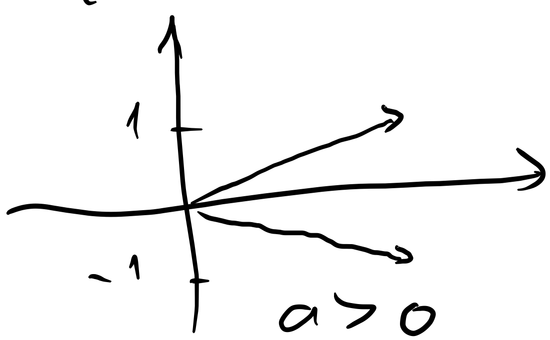
$B(x,y) = x^T M y$

Q-ort $B(e_i, e_j) = 0$ ha $i \neq j$.

$Q(x,y) = 2xy$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$B\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix}\right) = ad + bc = 0$

$\begin{bmatrix} a \\ 1 \end{bmatrix} \begin{bmatrix} a \\ -1 \end{bmatrix}$ Q-ortog $a \neq 0$ esetén F $\rightsquigarrow$ bázis.



$Q(a,1) = a > 0$

$Q(a,-1) = -a < 0$ indef.

$U = \left\langle \begin{bmatrix} a \\ 1 \end{bmatrix} \right\rangle = V$

a-tól függ.

$W = \{0\}$ IF

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} ? \\ 0 \end{bmatrix}$

nem F

$W = \langle e_i \mid Q(e_i) = 0 \rangle$ függ az e_i-k választásától.

Áll. $Q(e_i) = 0 \iff B(e_i, v) = 0 \quad \forall v.$

$v = \sum \lambda_i e_i$

$B(e_i, v) = \sum \lambda_i \underbrace{B(e_i, e_j)}_{\substack{0 \text{ ha } i \neq j \\ 0 \text{ ha } i = j}} = 0.$ $\quad Q(e_i) = 0$

$B(w_1, v) = 0$

$B(w_2, v) = 0 \quad \Rightarrow \quad B(w_1 + w_2, v) = 0$

$B(\lambda w_1, v) = 0$ ✓

$\overline{W} = \{ w \mid \forall v \;\; B(w, v) = 0 \}$

HF, Freud: lehet találni Q-ortog bázist $\forall$ elemhez.

VII/8 $\quad B$ szimm, valós.

$\forall \neq 0$ vektor $\in$ B-ortog bázis

$\iff$ B nem indefinit.

Q indefinit $\quad \lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2$

$\lambda_1 > 0 \quad \lambda_2 < 0$

$$\begin{bmatrix} \sqrt{-\lambda_2} \\ \sqrt{\lambda_1} \\ 0 \end{bmatrix} = e \qquad \lambda_1 \cdot (-\lambda_2) + \lambda_2 (-\lambda_1) + \lambda_3 \cdot 0 = 0$$

$Q(e) = 0$

$e \in Q$-ortog bázis

$\Rightarrow B(e, v) = 0 \quad \forall v.$

$$B\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right) = \lambda_1 \underset{\sqrt{-\lambda_2}}{\underset{|}{x_1}} \underline{y_1} + \lambda_2 \underset{\sqrt{\lambda_1}}{\underset{|}{x_2}} \underline{y_2} + \lambda_3 \underset{0}{\underset{\shortparallel}{x_3}} y_3 \neq 0$$

Nem indefinit

I rend-alg.

$\lambda_1 \sqrt{-\lambda_2}\, y_1 + \lambda_2 \sqrt{\lambda_1}\, y_2 \neq 0 \quad \exists y_1, y_2$

$y_2 = 1, \; y_1 = 0.$

$$\left|((B(v_i, v_j)))\right| = 0 \iff v_1, \text{——}, v_n$$ 7

VII/9

B nem elfajuló

$$((B(v_i, v_j))) = A^* M A \quad \text{HF.}$$

$[B] = M \qquad B(u, v) = u^* M v.$

$$A = [v_1, \text{——}, v_n]$$

$$\underbrace{\begin{array}{c} \scriptstyle i \\ \boxed{0 \; 1 \; 0} \end{array}}_{= e_i} \begin{bmatrix} & & \\ & & \\ & B & \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_j = i. \text{ sor } j. \text{ elem.}$$

$$e_i^* A^* M A e_j = (Ae_i)^* M (Ae_j) = B(\underbrace{Ae_i}_{= v_i}, \underbrace{Ae_j}_{= v_j}).$$

$$\text{Gram-det} = \det A^* M A =$$

$$= \det M \cdot \det A^* \cdot \det A.$$

$$\det A^* = \overline{\det A}$$

Ha $\det M \neq 0$

Gram-det $= 0 \iff \det A = 0 \iff v_i$ $\boxed{\text{ÖF}}$

B elfajuló $\Rightarrow$ $\det[B] \neq 0$.

$$B(u,v) = u^* M v$$

v cdott $\forall u$-ra $B(u,v)=0$

$$\Leftrightarrow \underbrace{(u^* M)}_{= (Mu)^*} v = 0 \quad \forall u.$$

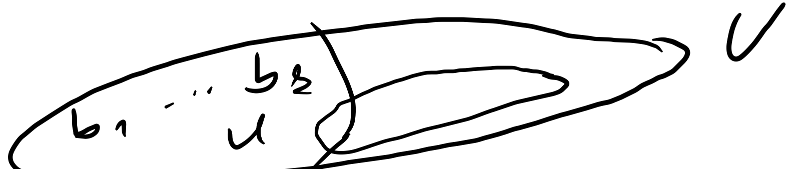
$$(Mu)^* \perp v$$

u $\forall$ vektor

M nem lehet injektív.

$\Leftrightarrow$ $\det M = 0$.

VII/10.



$M = [B]$

$\dim V = n$

$\dim U = k$.

$$0 = \begin{bmatrix} B(b_1,v) \\ \vdots \\ B(b_k,v) \end{bmatrix} = \begin{bmatrix} b_1^* M v \\ \vdots \\ b_k^* M v \end{bmatrix}$$

$\uparrow C$

lin. lek. v $V \to K^k$. $\mathrm{Im}\, C \leq k$-dim.

$\Rightarrow$ $\mathrm{Ker}\, C \geq n-k$-dim, $\mathrm{Ker}\, C = U^\perp$. ✓

(2) $U \subseteq (U^\perp)^\perp$ triviális

$u \in U \qquad U^\perp = \{w \mid B(w,u) = 0\}$

u jó $\forall w$-ben $\Rightarrow u \in (U^\perp)^\perp$.

Ha B nem elfajuló, és már tudjuk, hogy (1)-ben $=$ van $\Rightarrow \dim U = \dim U^{\perp\perp}$

$=$ egyenlőség van.

Kell B nem elfajuló $\Rightarrow \dim U^\perp = n-k$.

$$C: v \longmapsto \begin{bmatrix} B(b_1, v) \\ \vdots \\ B(b_k, v) \end{bmatrix}$$

$\Uparrow$

Az kell, hogy C ráképez K^k-ra.

$\mathrm{Im}\, C$ valódi altér

$\Rightarrow \exists$ rá $\perp$ vektor $\neq 0$. $\begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \end{bmatrix} \neq 0$.

$\lambda_1 B(b_1, v) + \ldots + \lambda_k B(b_k, v) = 0 \quad \forall v$

$= B(\underbrace{\lambda_1 b_1 + \ldots + \lambda_k b_k}, v) \quad \forall v \in V$. B nem elf

$\Rightarrow \forall \lambda_i = 0$ ξ.

VII/11.

$u \perp_B v$ minimum.

$[B] = M$

$\underbrace{u^* M v} = 0 \iff v^* M u = 0$

$\iff (v^* M u)^* = 0$

$\qquad\qquad \| $

$\underbrace{u^* M^* v} = 0$

$u^* M \perp v \iff u^* M^* \perp v.$

$\langle v \rangle^\perp$ n-1-dim altér. $Mu \in \Rightarrow M^* u$ is.

u fix $\langle u^* M \rangle^\perp = \langle u^* M^* \rangle^\perp$

n-1-dim n-1-dim.

$\Rightarrow \langle u^* M \rangle = \langle u^* M^* \rangle$

$u^* M \parallel u^* M^*$ sőt $u^* M = \lambda u^* M^* \quad \lambda \neq 0.$

$M^* u = \lambda M u$

$Mv = \lambda M^* v$

$Mw = \mu M^* w$

$\langle Mv, w \rangle = \langle v, M^* w \rangle =$

$= \langle v, \mu M w \rangle = \mu \langle M^* v, w \rangle =$

$= \underline{\mu \lambda} \langle Mv, w \rangle \qquad Mw \perp v$

van $\lambda \mu = 1.$

$\lambda \mu = 1.$

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad M^* = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \end{bmatrix}$$

$a = \lambda a$
$c = \lambda b$

Ha $a \neq 0$
$\Rightarrow \lambda = 1$
$\Rightarrow c = b$
kész.

Ha $d \neq 0$: ugyanez $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$-val.

Ha $a = d = 0$

$$\begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 0 & c \\ b & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = 1$
kész
$\lambda = -1$ alternáló

$$\begin{bmatrix} b \\ c \end{bmatrix} = \lambda \begin{bmatrix} c \\ b \end{bmatrix}$$

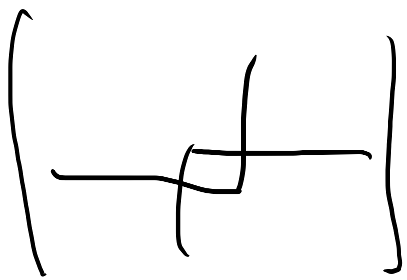
$b = \lambda c$
$c = \lambda b$

$b = \lambda^2 b$
$c = \lambda^2 c$
(vagy $b = c = 0$).

$\Rightarrow \lambda^2 = 1$

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ \\ \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

3×3-asnál $\Rightarrow$ van nincs megoldás
van általános megoldás



H.F.

$\mathbb{C}$ $\quad M^* = -iM \qquad |-i| = 1$

$$\underbrace{\begin{bmatrix} 1+i & 2 \\ 2i & -1-i \end{bmatrix}} = -i \begin{bmatrix} 1-i & -2i \\ 2 & -1+i \end{bmatrix}$$

$B(u,v) = 0 \qquad u^* M v = 0$

$\Rightarrow 0 = (u^* M v)^* = v^* M^* u = -i \underbrace{v^* M^* u}_{=0} \qquad v \perp_B u$

$= ,$