$U \oplus W$. $U + W = \{ u + w \mid u \in U,\ w \in W \}$

$U, W \le V$ HA $U \cap W = \{0\}$

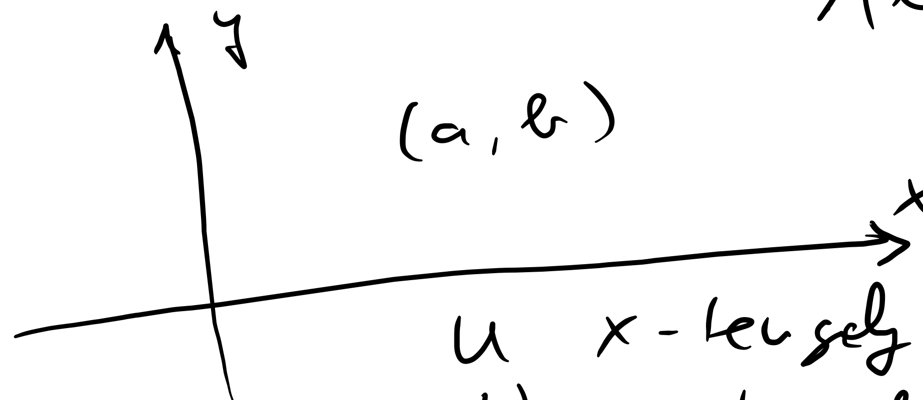
akkor $U + W$ lesz $U \oplus W$.

$U, W \le V$ $\longrightarrow$ $U + W \longleftrightarrow U \times W$

Ha U, W vektorterek T fölött | $u + w$ egyért. $\rightsquigarrow u, w$.

Direkt összeg.

Általában $U \times W$ Direkt szorzat vektortér T fölött



sík

U x-tengely $\mathbb{R}$ $\mathbb{R}$ fölött

W y-tengely $\mathbb{R}$ – " –

sík $= \{ (a, b) \mid a \in U,\ b \in W \}$.   sorvektorokként is tekinthetők.

$(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$

$\lambda (a, b) = (\lambda a, \lambda b)$

Descartes szorzat: halmazoké $\mathbb{R} \times \mathbb{R}$.

$U, W \le V$

$U \times W \longmapsto V$

$(u, w) \xmapsto{A} u + w \in V.$

HF ez lineáris leképezés.

$\mathrm{Ker}\, A = \{(u, w) \mid u + w = 0\}$

$\mathrm{Im}\, A =$
$= \{u + w \mid u \in U, w \in W\}$
$= U + W$

$B : U \times W \to V \qquad (u, w) \xmapsto{B} u - w. \quad$ (lin.)

$\mathrm{Ker}\, B = \{(u, w) \mid u - w = 0_V\}$

azaz $u = w. \in W$

$\mathrm{Ker}\, B = \{(u, u) \mid \cancel{u \in U \text{ és } u (= w) \in W}\} \hat{=} U \cap W$

$u \in U \cap W.$

$\mathrm{Im}\, B = \{u - w \mid u \in U \text{ és } w \in W\} = \underline{U + W}$

$u - w = \underset{\in U}{u} + \underset{\in W}{(-w)}$ $\forall w$-ből

megkapunk.

dim-tétel: $\dim(U + W) + \dim(U \cap W) = \dim U \times W.$

$\qquad \overset{HF}{=} \dim U + \dim W.$

$\mathrm{Ker} A \cong U \cap W$

$u \mapsto (u, -u).$

HF:

$U \times W$ bázisa $(b_1, 0_W), \dots, (b_n, 0_W), (0_U, c_1) \dots \dots (0_U, c_k)$

U $\quad b_1 - b_n$ $\quad$ B

W $\quad c_1 - c_k$ $\quad$ B

IV/10 $\quad F_k$ k-adik Fibonacci-szám

$$\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} \Big/ \sqrt{5} \text{ -höz}$$

legközelebbi egész.

Mert explicit képlet: lásd feladatsor,

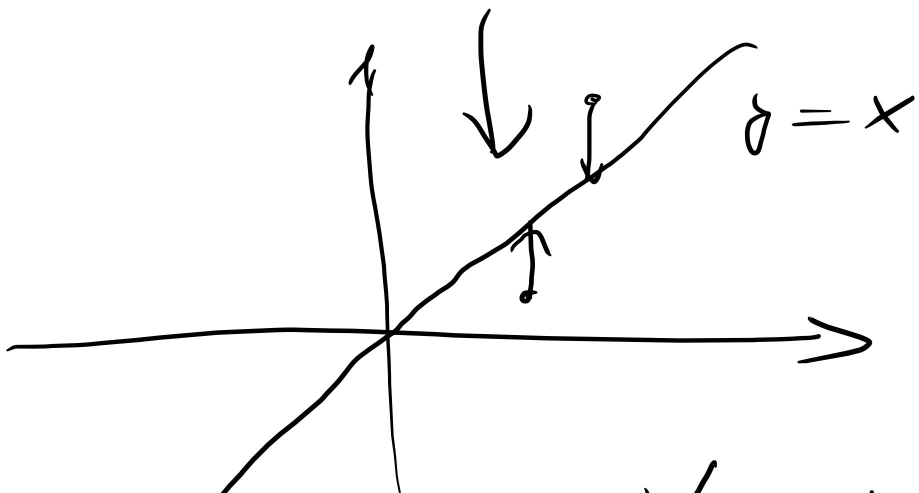
és $\left(\frac{1-\sqrt{5}}{2}\right)^{k+1} \longrightarrow 0$ $\qquad k \to \infty$ $\qquad \left|\frac{1-\sqrt{5}}{2}\right| < 1.$

ez idő után $< 1/2$

IV/11 A $\in \mathrm{Hom}\,V$ PROJEKCIÓ (vetítés) ha $A^2 = A$.

↗ idempotens.

Pl.



$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x \\ x \end{pmatrix}$

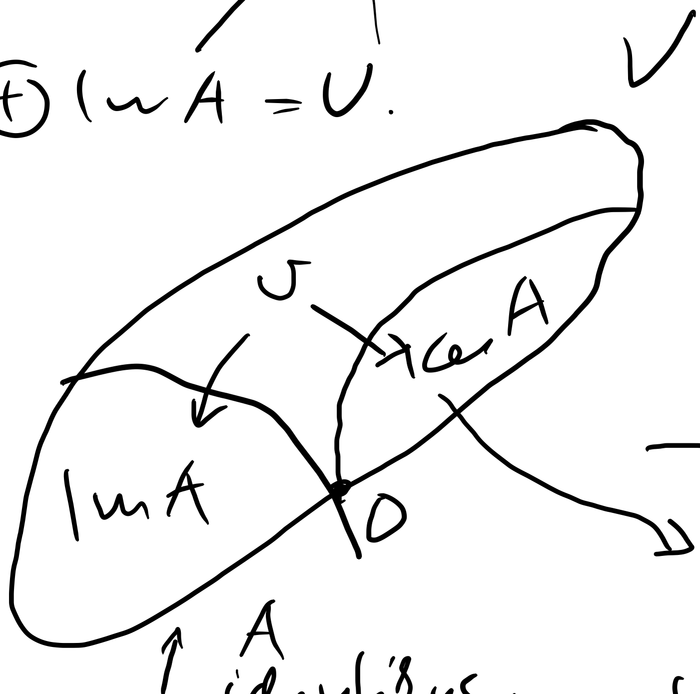
Im $y = x$ egyenes

Ker y-tengely

$\mathrm{Ker}A \oplus \mathrm{Im}A = V$.

Im vetítés képe (ahová)

Ker iránya (ahonnan).



$A^2(v) = A(v)$. $\forall v$

$A(A(v)) = A(v)$

$A(\underbrace{A(v) - v}_{\in \mathrm{Ker}A}) = 0$

$v = \underset{\in \mathrm{Im}}{A(v)} + \underset{\in \mathrm{Ker}}{(v - A(v))}$.

$\mathrm{Im}A + \mathrm{Ker}A = V$.

$v \in \operatorname{Im} A \cap \ker A$

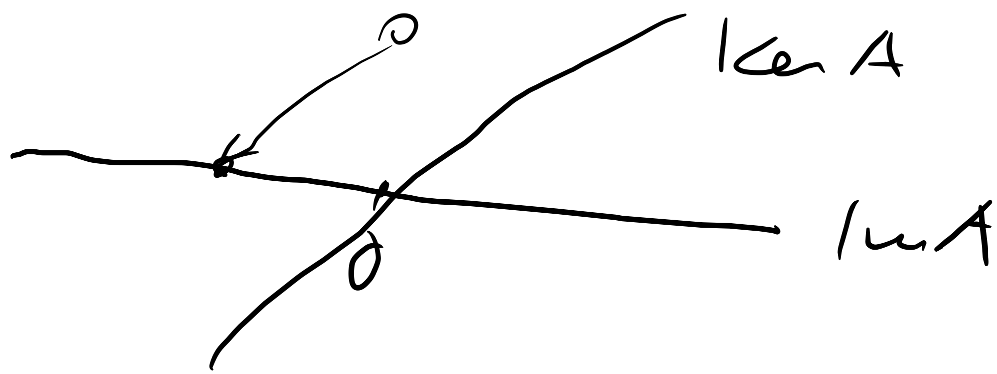
$A(v) = 0$

$\Rightarrow A(v) = v$ weil $A(A(w)) = A(w)$

$\Rightarrow 0 = v$. ✓

Spec $\ker A = \{ v - A(v) \mid v \in V \}$.

HF.



d'où: Ha $\dim \operatorname{Im} A = 1$ $\Rightarrow \dim \ker A = 1$.

$\dim \operatorname{Im} A = 2 \Rightarrow A = \mathrm{id}$
$\Rightarrow$ [illegible] $\mathbb{R}^2$.

$\dim \operatorname{Im} A = 0 \Rightarrow \dim \ker A = 2 \Rightarrow \ker A = \mathbb{R}^2$.
$\Rightarrow A = 0$.

$$\left[\begin{array}{ll} U \times W \longrightarrow U & \qquad U \times W \longrightarrow W \\ (u,w) \longmapsto u & \qquad (u,w) \longmapsto w. \end{array} \right]$$

IV/12. $A \in \mathrm{Hom}\, U$ véges dim-ú U

$\exists k \quad U = \mathrm{Im}\, A^k \oplus \mathrm{Ker}\, A^k$.

HF: $U \supseteq \mathrm{Im}\, A \supseteq \mathrm{Im}\, A^2 \supseteq \dots \supseteq 0$

$0 \subseteq \mathrm{Ker}\, A \subseteq \mathrm{Ker}\, A^2 \subseteq \dots \dots \subseteq U$.

Előbb-utóbb $\dim \mathrm{Im}\, A^k \underbrace{= \mathrm{Im}}_{\dim} A^{k+1}$.

$\underset{\text{altér}}{\Longrightarrow} \quad \mathrm{Im}\, A^k = \mathrm{Im}\, A^{k+1}$

dim-tétel $\quad \mathrm{Ker}\, A^k = \mathrm{Ker}\, A^{k+1}$

Áll. $\underbrace{\mathrm{Im}\, A^k = \mathrm{Im}\, A^{k+1}}_{(\supseteq)} \Longrightarrow \mathrm{Im}\, A^k = \mathrm{Im}\, A^{k+n}$ $\forall n \geq 1$.

Ha $v = A^k(w) \Longrightarrow v = A^{k+1}(u)$
$\exists w \qquad \exists u$

$v \in \operatorname{Im} A^k = \operatorname{Im} A^{k+1}$

$v = A^k \cdot w \Rightarrow v = A^{k+1} u$

$A^k u = A^{k+1} u'$

$\in \operatorname{Im} A^k = \operatorname{Im} A^{k+1}$

$v = A^{k+1} u = A^{k+2} u' \in \operatorname{Im} A^{k+2}$. stb.

HF zöttvetlenül ugyanezt megteremzve.

$\{A^2 = A \Rightarrow \operatorname{Im} A = \operatorname{Im} A^2$ és $\operatorname{Ker} A = \operatorname{Ker} A^2$

$V = \operatorname{Im} A \oplus \operatorname{Ker} A$. $\}$

$\operatorname{Im} A^k = \operatorname{Im} A^{k+1} \Rightarrow \operatorname{Im} A^k = \operatorname{Im} A^{2k}$

(és $\operatorname{Ker} A^k = \operatorname{Ker} A^{2k}$).

$B = A^k$. $\operatorname{Im} B^2 = \operatorname{Im} B \quad \operatorname{Ker} B^2 = \operatorname{Ker} B$.

$\overset{1.}{\Rightarrow} \quad V = \operatorname{Im} B \oplus \operatorname{Ker} B$.

$\operatorname{Ker} B^2 = \operatorname{Ker} B$, $\operatorname{Im} B^2 = \operatorname{Im} B$

$V = \operatorname{Im} B + \operatorname{Ker} B$ és $\operatorname{Im} B \cap \operatorname{Ker} B = 0$.

$v \in \operatorname{Im} B \cap \operatorname{Ker} B \overset{?}{\Rightarrow} v = 0$

$$\left.\begin{array}{l} B(v) = 0 \text{ és} \\ \exists w \quad B(w) = v \end{array}\right\} \Rightarrow B^2(w) = B(v) = 0.$$

$\Rightarrow w \in \operatorname{Ker} B^2 = \operatorname{Ker} B \Rightarrow \underset{v}{\underset{\shortparallel}{B(w)}} = 0$

$\Rightarrow v = 0$.

$v \in V$ $\quad B(v) \in \operatorname{Im} B = \operatorname{Im} B^2$

$\Rightarrow B(v) = B^2(w) \Rightarrow B(\underbrace{v - B(w)}_{\in \operatorname{Ker} B}) = 0$

$v = \underbrace{B(w)}_{\in \operatorname{Im} B} + \underbrace{(v - B(w))}_{\in \operatorname{Ker} B}$.

TV/13

$M^n \to 0$ (elemenként)

$\iff$ M $\forall$ s.é.-e $|\ | < 1$. $M \in \mathbb{C}^{n\times n}$

Biz. $S^{-1} M^n S = (S^{-1} M S)^n$ S fix

$M^n \to 0 \iff \searrow 0 \iff (S^{-1} M S)^n \to 0$

Elég, ha M Jordan-alakú.

elég blokkonként.

$$\begin{bmatrix} \lambda & \ddots & 0 \\ 1 & \ddots & \\ 0 & 1 & \lambda \end{bmatrix}^n_{r\times r} = \text{explicit képlet}$$

$\lambda^n, \lambda^{n-1}, \ldots, \lambda^{n-r+1} \to 0$

$\binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{r}$ ha $|\lambda| < 1$.

ha van $|\lambda| \geq 1$ $\lambda^n \not\to 0$.

TU /14 $\quad f = f_1 + \ldots + f_n$

periodikusak.

FOURIER-sor

$$f(x) = \sum_{i=0}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

f múzsikája $\longrightarrow$ $f : \mathbb{R} \longrightarrow \mathbb{R}$

idő $\qquad$ hangerő

---

f n-edfokú pol. $\qquad f = f_1 + \ldots + f_n$

$s_1 \qquad s_n$

s szerint periodikus f $\qquad$ a periódusok.

$$f(x+s) = f(x) \quad \forall x\text{-re}.$$

$$f(x+s) - f(x) = g(x)$$

$$D_s : f \longmapsto g.$$

$D_r D_s(f) = D_s D_r(f) =$

$r, s > 0$
valós

$= f(x+r+s) - f(x+r) - f(x+s) + f(x)$.

HF.

$f = f_1 + \dots + f_n$ HF D_s lineáris

$(D_s(f_1+f_2) = D_s(f_1) + D_s(f_2))$

$D_{s_1} D_{s_2} \dots D_{s_n}(f) =$

$= \sum_i D_{s_1} S_{s_2} - D_{s_n}(f_i)$.

$D_{s_i}(f_i) = 0$ mert f_i s_i-periodikus.

$\Downarrow$

$D_{s_1} \dots D_{s_{i-1}} D_{s_{i+1}} \dots D_{s_n} D_{s_i}(f_i) = 0$.

$\Rightarrow$ $D(f) = 0$ akkor, ha $f = n$ db periodikus összege

$D = D_{s_1} \dots D_{s_n}$.

f n-edfokú $\Rightarrow$ $D_s(f)$ $n-1$ fokú.

(elő_f : n-edik : 1. fokú, a szám $\neq 0$).

$$(x+s)^n - x^n = x^{n-1}\cdot\binom{n}{1} + \text{alacsonyabb fokúak}$$

$(x+s)^{n-1} - x^{n-1}$ $\quad$ x^{n-1} kiesik

$$f(x) = a x^n + \dots$$

$D(f)$-ben x^{n-1} együtthatója $a\binom{n}{1}$

x^n nincs.

$\Rightarrow D(f)$ foka $n-1$.