3/(c) $\lg 2,\ \lg 3,\ \lg \underset{2\cdot 3}{\underset{\shortparallel}{6}} = \lg 2 + \lg 3$ öf.

R or Q fölött.

$\lg 2,\ \lg 3,\ \lg 5$

$\mathbb{R}$ $\forall$ 2 valós szám öf $\qquad r, s \in \mathbb{R}$

$s \cdot r - r s = 0$

Ha $s = r = 0 \qquad 1 \cdot r = 0$

$\mathbb{Q}$ (F) számelm. alaptétel. $\qquad q_2 q_3 q_5 \neq 0$

$$p_2/q_2 \lg 2 + p_3/q_3 \lg 3 + p_5/q_5 \lg 5 = 0$$

$$10^{\wedge(\ )} \qquad 2^{p_2/q_2} \cdot 3^{p_3/q_3} \cdot 5^{p_5/q_5} = 1 \qquad (\ \ )^{q_2 q_3 q_5}$$

$$2^{p_2 q_3 q_5} \cdot 3^{p_3 q_2 q_5} \cdot 5^{p_5 q_2 q_3} = 1$$

sz. e. alaptétele $\forall$ kitevő 0.

$(x-a)(x-b) \quad (x-b)(x-c) \quad (x-c)(x-a)$

$$\begin{array}{l|ccc} 1 & ab, & bc, & ca \\ x & a+b, & b+c, & a+c \\ x^2 & 1 & 1 & 1 \end{array}$$

$$\alpha \downarrow + \beta \downarrow + \gamma \downarrow = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \alpha = \beta = \gamma = 0.$ F

$$\begin{vmatrix} 1 & 1 & 1 \\ a+b & b+c & c+a \\ ab & bc & ca \end{vmatrix} \neq 0 \quad \text{pro } a, b, c$$

(platí též.

VANDERMONDE.

Ach. [illegible] platí [illegible] u pol. 5-k, [illegible]

$\pm(a-b)(b-c)(c-a) \neq 0$

první řádek x^2
2. řádek $-x$ } + 3. sloupec

3. sl. $(x-a)(x-b) \quad (x-b)(x-c) \quad (x-c)(x-a)$

$x = a, b, c$ ukáže [illegible] řešit lin. fce. H.

(6.)

$1, \sqrt{2}$ (F) $\mathbb{Q}$ felett

$$a + b\sqrt{2} = 0 \implies \sqrt{2} = -a/b \quad \lightning$$

irrac rac

$$\Downarrow$$

$$a = 0 \iff b = 0$$

$$\sqrt{3} = a + b\sqrt{2} \quad / \text{négyzetre}$$

$$0 = \underbrace{-3 + (a^2 + 2b^2)}\, + \underbrace{(2ab)}\sqrt{2}$$

NEM.

$$\implies 2ab = 0 \quad \text{és} \quad a^2 + 2b^2 = 3$$

$$b = 0 \implies a = \sqrt{3} \quad \lightning \quad \sqrt{3} \text{ irrac}$$

$$a = 0 \qquad \sqrt{3} = b\sqrt{2} \implies \sqrt{6} = 2b \quad \lightning$$

irrac. rac

$$a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} = 0 \qquad A + B\sqrt{2}$$

$$\sqrt{3}(c + d\sqrt{2}) \qquad A, B \in \mathbb{Q}$$

$$\sqrt{3} = \frac{-a - b\sqrt{2}}{c + d\sqrt{2}} = \frac{(-a - b\sqrt{2})(c - d\sqrt{2})}{c^2 - 2d^2} \quad \lightning$$

Tehát $c + d\sqrt{2} = 0 \implies c = d = 0 \implies a = b = 0$

$$\left\{\begin{array}{l} \alpha c + \beta b + \delta d = 0 \quad \exists\, \alpha, \beta, \delta \neq 0 \\ a, b, c \text{ (F)} \Rightarrow \delta \neq 0 \end{array}\right.$$

$\Rightarrow$ d-t ki lehet fejezni

$$\left.\begin{array}{l} d = \alpha_1 a + \beta_1 b \\ d = \alpha_2 a + \gamma_2 c \\ d = \beta_3 b + \gamma_3 c \end{array}\right\}$$

Hasonlóan d három felírása a, b, c-vel.

$$\Rightarrow \left.\begin{array}{l} \alpha_1 = \alpha_2 \\ \beta_1 = 0 \\ \gamma_2 = 0 \end{array}\right|$$

a, b, c (F) $\Rightarrow$ d felírás

$\Leftarrow$ egyértelmű.

stb. $\forall$ együttható 0 $\Rightarrow$ $d = 0$.

---

8. n páros pl. $n = 4$ ÖF

$$v_1 + v_2 \qquad v_2 + v_3 \qquad v_3 + v_4 \qquad v_4 + v_1$$

$+$

$=$

8.

Ha n ptlan : H

ellenpéldázhatjuk ez.

$\alpha_1 + \alpha_2 = 0$ $\quad \alpha_1 = -\alpha_2$

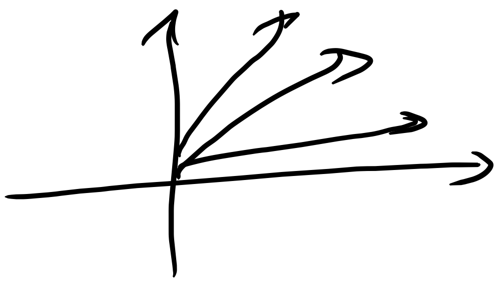
$\alpha_2 + \alpha_3 = 0$ $\quad \alpha_2 = -\alpha_3$

$\vdots$

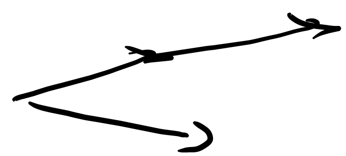
Viszont n ptlan

$\alpha_1 = -\alpha_1 \Rightarrow \alpha_1 = 0.$

9.



ÖF UZ H



0 összeg.

HF

10.

$0v = 0 \qquad 1v = v \qquad \mathbb{F}_2 = \mathbb{Z}_2 = \{0, 1\}$

NEM vektortér

$(1 +_2 1)\, v = 1v + 1v$

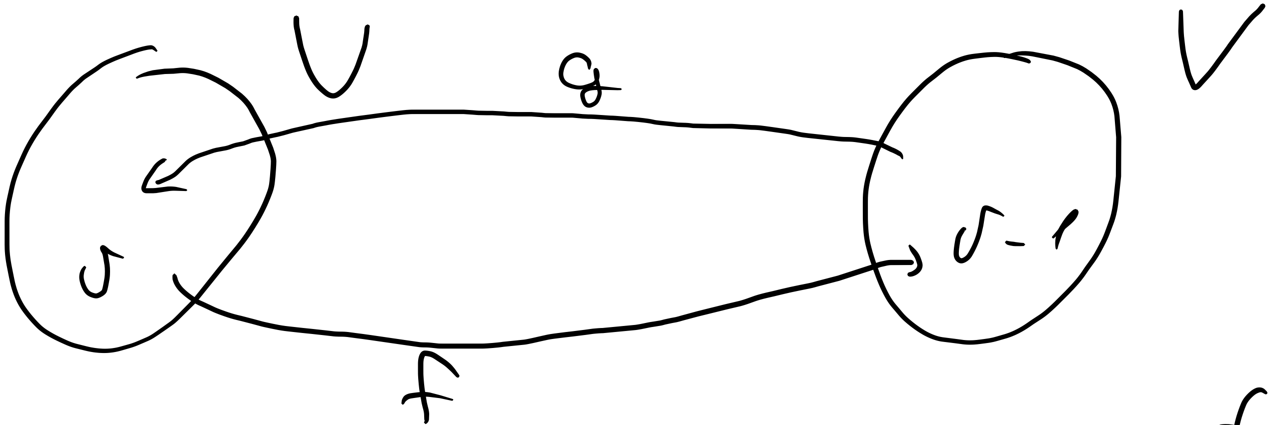
$\overset{\shortparallel}{0}$ $\quad$ $\mathbb{R}$-beli $+$

$$\boxed{\begin{matrix} 0 \neq 2v \\ 0 \neq 2 \end{matrix}}$$

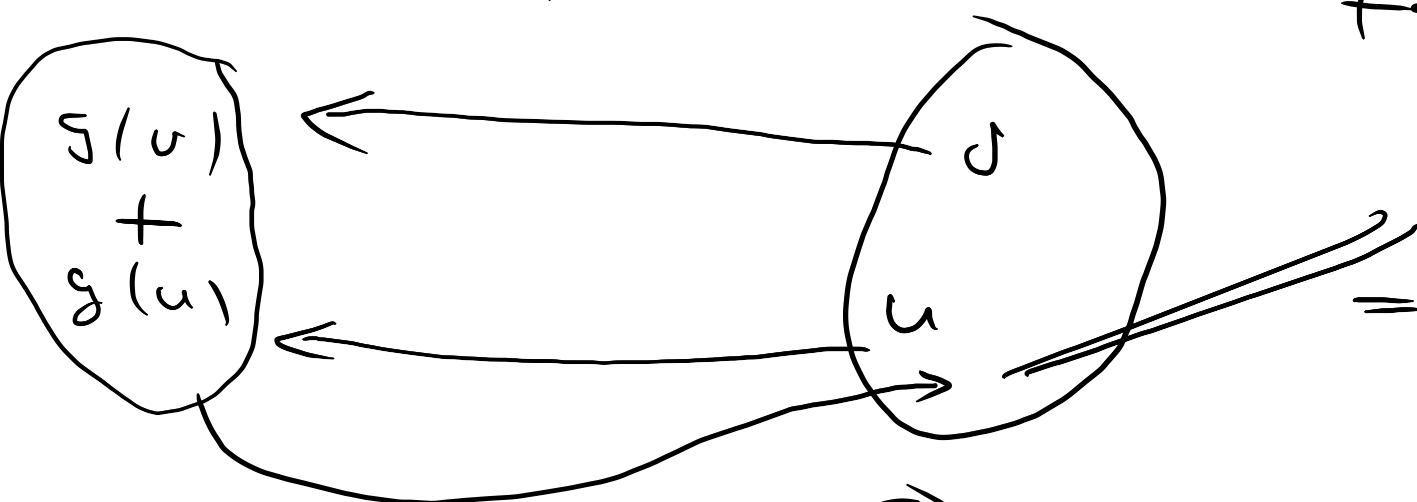
11.

$$u \oplus v = u + v - 1$$
$$\lambda \odot v = \lambda v - \lambda + 1.$$

KONJUGÁLÁS



$$f(v) = v - 1$$
$$g(v) = v + 1.$$
$$f \circ g = id = g \circ f$$



$$f(g(u) + g(v)) =$$
$$= u + 1 + v + 1 - 1$$
$$= u + v + 1$$

$$g(f(u) + f(v)) =$$
$$= u - 1 + v - 1 + 1 = \underline{\underline{u + v - 1}}$$

HF λ-szorzásra is jó.

Altér $U \subseteq V$ vektortér T fölött

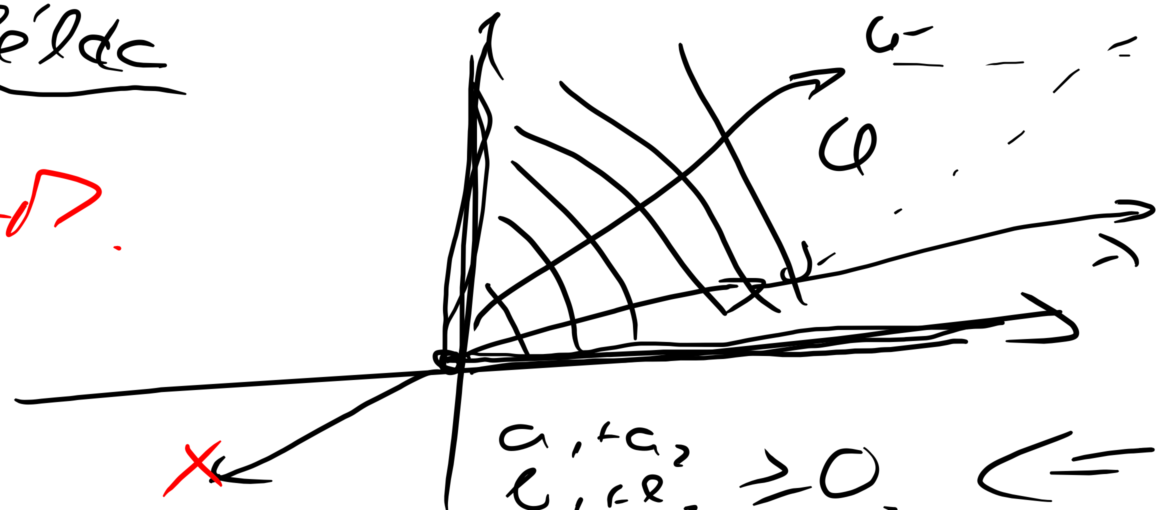
altér, ha $u_1, u_2 \in U \Rightarrow u_1 + u_2 \in U$

$u \in U \Rightarrow \lambda u \in U \quad \forall \lambda \in T$.

$0 \in U \quad (\Leftrightarrow U \neq \emptyset)$.

14. Példa

NEM.

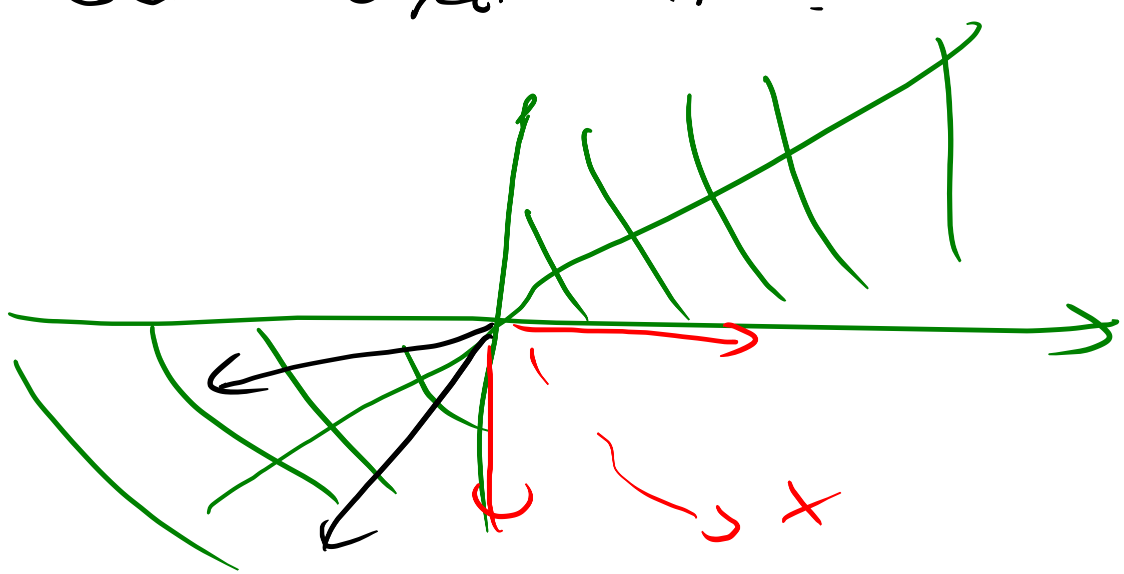


$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$a_1, a_2 \geq 0$

$a_1 + a_2$
$e_1 + e_2 \geq 0. \Leftarrow \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \mid e_1, e_2 \geq 0$

λ lehet negatív is!



NEM

(1) $\leq 10.$ fokú és 0 — I ✓

(2) $\geq 10.$ fokú és 0 — N $\quad x^{12}+x \;+\; (-x^{12})$

(3) páros fokú és 0 — N

(4) $\sum a_i x^{2i}$ — ~~N~~ I ✓ $\quad$ 0 = üres összeg!

$\mathbb{R}$ felett!

---

(5) 3×3 $\forall$ elem rac — I $\quad \mathbb{Q}$ felett N $\quad \mathbb{R}$ felett $\sqrt{2}$-vel szorozva

(6) $\det = 0$ — ~~I~~ N $\quad \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$ ✓

(7) van 2 azonos — N $\quad \begin{pmatrix}1&1&2\\3&4&5\\6&7&8\end{pmatrix} + \begin{pmatrix}1&2&3\\4&5&6\\7&8&8\end{pmatrix} \notin$

(8) $a_{11}=a_{12}$ — I ✓

(9) $\Sigma = 0$ — I ✓

(10) $\Pi = 0$ — N $\quad \begin{pmatrix}0&1&1\\1&1&1\\1&1&1\end{pmatrix} + \begin{pmatrix}1&0&1\\1&1&1\\1&1&1\end{pmatrix} \notin$

(11) $\Sigma = 3$ — N $\quad \lambda$-szoros nem jó $\quad$ 0-mátrix $\notin$.

(12) $\sum a_{ij}^2 = 0$ — N? $\quad \mathbb{R}$: $\Rightarrow a_{ij}=0 \;\forall i,j$

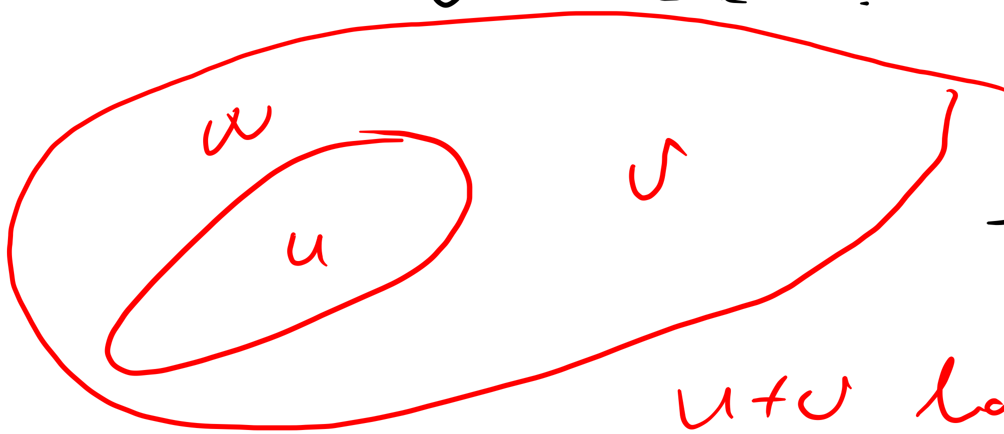
$\mathbb{R}$ elemű

I: csak a 0 mátrix $\quad \mathbb{C}$ felett N.

HF.

INDIREKT!

15. (1)



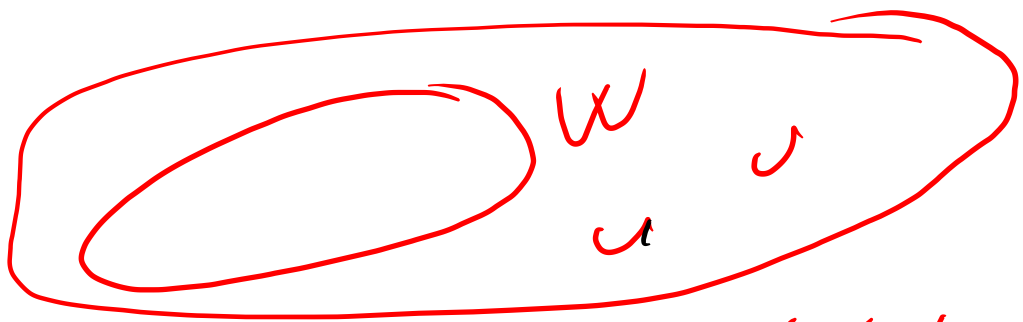
$$\begin{array}{rl} & u+v \in W \\ + & -u \in W \\ \hline & v \in W \text{ ↯} \end{array}$$

$u+v$ hol?

$\Longrightarrow$ KINT.

$u+v \notin W$.

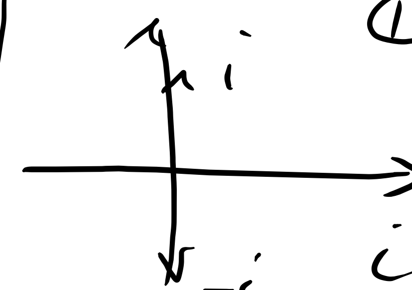
15. (2)



$u+v$ hol?

Példákat, ha azt gondolják, hogy kint is, bent is lehet.

$V = \mathbb{R}$



$\mathbb{C}$ $\mathbb{R}$

$i + (-i)$ bent

$i + i + 1 \notin$ bent.

15/3-4 HF.

$\langle v_1, \dots, v_n \rangle$ a legszűkebb, ami altér.
(a lehető legkevesebb vektort vesszük be)

$\lambda_1 v_1 + \dots + \lambda_n v_n$ biztos kell
↗ összes így altért alkot.

$x \in \langle x^2-1, x^2-2, 3x+2 \rangle = U$ ✓ IGEN

$x = \alpha(x^2-1) + \beta(x^2-2) + \gamma(3x+2)$ $\alpha, \beta, \gamma \in \mathbb{R}$ ismeretlen

$$\left.\begin{array}{lrl} 1 & 0 = & -\alpha - 2\beta \\ x & 1 = & 3\gamma \\ x^2 & 0 = & \alpha + \beta \end{array}\right\}$$

$\exists$ a mo. ($\exists$-ilos sol.)

Lin. egy.rsz.

HF.

II. mo.

$$\begin{array}{r} x^2 - 1 \in U \\ -\ x^2 - 2 \in U \\ \hline 1 \in U \end{array} \qquad \begin{array}{r} 1 \in U \\ 3x + 2 \in U \\ \hline +3x \in U \end{array} \Rightarrow 1/3(3x) = x \in U$$

$3x = 3x + 2 - 2 \cdot 1$

$$\langle v_1, v_2, v_2 \rangle \overset{?}{=} \langle u_1, u_2 \rangle$$

$$\underbrace{v_1, v_2, v_3 \in \langle u_1, u_2 \rangle}$$

$$\Rightarrow \langle v_1, v_2, v_3 \rangle \subseteq \langle u_1, u_2 \rangle$$

legszűkebb altér, amiben eleme v_1, v_2, v_3

altér eleme v_1, v_2, v_3

$$\overset{?}{\supseteq}$$

még $u_1, u_2 \overset{?}{\in} \langle v_1, v_2, v_3 \rangle$

HF. sima módszerrel.