$3x^3+4x^2+4x+1$, rac. gyökök.

p/q rac gyök, $(p,q)=1$

$\Rightarrow p \mid 1$, $q \mid 3$

$p=\pm 1$, $q=\pm 1, \pm 3$.

$p/q = \pm 1, \pm 1/3$. Poz. gyök nincs.

-1 vagy $-1/3$

	3	4	4	1
$x=-1/3$	3	3	3	$\underline{\underline{0}}$

$3x^2+3x+3$

nincs valós gyök

$$\left(x-\left(-1/3\right)\right)\left(3x^2+3x+3\right) = (3x+1)(x^2+x+1)$$

$-1/3$ az egyetlen rac. gyök, egyszeres.

$\boxed{2.4.20}$

T/15* $f \in \mathbb{Z}[x]$.

$f(10) = 400$, $f(14) = 440$, $f(18) = 520$

$f(x) - 400$ gyöke a 10

$f(x) - 400 = (x-10)\, g(x)$

$g \in \mathbb{Z}[x]$ a Horner elrendezésben csupa egész együttható keletkezik.

$f(14) - 400 = 440 - 400 = 40$

$\hookrightarrow (14-10)\, g(14)$

$40 = 4 \cdot g(14) \Rightarrow \boxed{g(14) = 10.}$

$x = 18$: $\underset{\overset{\shortparallel}{8}}{(18-10)}\, g(18) = f(18) - 400 = 520 - 400$
$= 120$

$\boxed{g(18) = 15.}$

$g(18) - g(14) = 5$
4-gyel nem osztható. ↯

Ha $f \in \mathbb{Z}[x]$
$a - b \mid f(a) - f(b)$
a, b egész.

$\boxed{NINCS f}$

I/16 2.4.26

$f \in \mathbb{Z}[x]$

4 kül helyen 5 az értéke

$\exists$-e $u \in \mathbb{Z}$ $f(u) = 12$?

$f(x) - 5$ gyöke a, b, c, d 4 kül egész.

$f(x) - 5 = (x-a)(x-b)(x-c)(x-d)\,g(x)$ $\in \mathbb{Z}[x]$.

$f(u) - 5 = \underbrace{(u-a)(u-b)(u-c)(u-d)}_{\text{4 kül egész}}\,g(u)$

$12 - 5 = \boxed{7}$

osztói $1, \underbrace{7, -1, -7}_{}$.

$u-a,\ u-b,\ u-c,\ u-d$

közül 3 db ± 1

pront kül ξ.

7, és -7 olcsosan vol enne!

$i^2 = -1$.

$(1+i)(3-2i) = 3-2i + i(3-2i) =$

$= 3-2i+3i-2\underbrace{i^2}_{=-1} = \underline{\underline{5+i}}$

$1/i = \frac{i}{i^2} = \frac{i}{-1} = -i$

$z = a+bi$ $\bar{z} = a-bi$ KONJUGÁLT

$z\cdot\bar{z} = (a+bi)(a-bi) = a^2-(bi)^2 = \underline{\underline{a^2+b^2}}$.

$|z| = \sqrt{a^2+b^2}$ $z\cdot\bar{z} = |z|^2$.

Osztásnál bővítünk a nevező konjugáltjával.

$\frac{1+i}{3-2i} = \frac{(3+2i)(1+i)}{(3+2i)(3-2i)} = \frac{3+2i+3i-2}{3^2+2^2} = \underline{\underline{\frac{1}{13}+\frac{5}{13}i}}$

(NEM $(2i)^2$!!)

$|z| = \sqrt{Re(z)^2+Im(z)^2}$.

$$\left|\frac{\overline{4+i}}{4+i}\right| = \left|\frac{4-i}{4+i}\right| = \left|\frac{(4-i)^2}{(4+i)(4-i)}\right| = \left|\frac{16-2\cdot 4 i+i^2}{4^2+1^2}\right| =$$

$$= \left|\frac{15-8i}{17}\right| = \left|\frac{15}{17} + \frac{-8}{17} i\right| =$$

$$\sqrt{\left(\frac{15}{17}\right)^2 + \left(\frac{-8}{17}\right)^2} = \underline{\underline{1}}.$$

$$\left|\frac{\overline{4+i}}{4+i}\right| \overset{6}{=} \frac{|\overline{4+i}|}{|4+i|} =$$

$$\overset{8.}{=} \underline{\underline{1}}.$$

$$|\overline{4+i}| = |4+i|$$

1. $z \cdot \bar{z} = |z|^2$
2. $\overline{z \pm w} = \bar{z} \pm \bar{w}$
3. $\overline{zw} = \bar{z}\,\bar{w}$
4. $\overline{z/w} = \bar{z}/\bar{w} \quad w \neq 0$
5. $|zw| = |z| \cdot |w|$
6. $|z/w| = |z|/|w| \quad w \neq 0$
7. $\bar{\bar{z}} = z$
8. $|\bar{z}| = |z|$
9. $z \in \mathbb{R} \Leftrightarrow z = \bar{z}.$

10. $|z^4| = |z|^4.$ 5 nicht.

$z = w \quad |z^2| = |z|^2$

$$|z^3| = |z^2 \cdot z| \overset{5}{=} |z^2| \cdot |z| = |z|^3$$

etc.

$$\left|\frac{(1+1.526i)^{100}}{(1-1.526i)^{100}}\right| \overset{6}{=} \frac{|(1+1.526i)^{100}|}{|(1-1.526i)^{100}|} =$$

10.

$$\frac{|1+1.526i|^{100}}{|1-1.526i|^{100}} = \left(\frac{|1+1.526i|}{|1-1.526i|}\right)^{100}$$

↳ ρ mind 1.

$(1+i)^2 =$ $\qquad (1+i)^{1241}$

$= 1^2 + 2\cdot 1\cdot i + i^2 = 1+2i-1 = \underline{\underline{2i}}$

$(1+i)^4 = ((1+i)^2)^2 = (2i)^2 = \underline{-4}$

$(1+i)^6 = (1+i)^2(1+i)^4 = 2i(-4) = \underline{\underline{-8i}}$

$(1+i)^8 = ((1+i)^4)^2 = (-4)^2 = 16$

$(1+i)^{12} = (-4)^3 = \underline{\underline{-64}}$.

4-vel osztható kitevőt könnyen!

$$1241 = 310\cdot 4 + 1$$

$$(1+i)^{1241} = \left[\underset{\overset{\parallel}{(-4)^{310} = 2^{620}}}{(1+i)^4}\right]^{310}\cdot(1+i) = 2^{620} + 2^{620}\, i$$

$x = i \;\vee\; x = -i$

$\Uparrow$

2. fokú egyenletek.

$x^2 = -1 \Leftarrow \pm i$, ill.

$x^2 + 1 = (x+i)(x-i) = 0$

$x^2 = -4 \Leftarrow \pm 2i$

$x^2 = -12 \Leftarrow \pm\sqrt{12}\; i$

Negatív valósból tudunk négyzetgyököt.

$r \geq 0 \qquad x^2 = -r \qquad x = \pm\sqrt{r}\; i.$

$$x^2 + 2x + 2 = 0 \qquad x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 2\cdot 2}}{2} = \frac{-2 \pm 2i}{2} = \underline{\underline{-1 \pm i}}$$

$$\underset{\text{főegyüttható}}{?}\,\Big(x - \big(\underset{\text{gyök}}{?}\big)\Big)\Big(x - \big(\underset{\text{gyök}}{?}\big)\Big)$$

$$1\big(x - (-1+i)\big)\big(x - (-1-i)\big) = \underline{\underline{(x+1-i)(x+1+i)}}.$$

$x^2+2ix-1=0 \qquad \dfrac{-2i\pm\sqrt{(2i)^2-4(-1)}}{2} = \dfrac{-2i\pm 0}{2} = -i$

$(x-(-i))^2 = (x+i)^2$

$-i$ kétszeres gyök.

$\sqrt{}$ algebrai módszere. $\pi/8$

$\sqrt{20i-21} = c+di \qquad c,d\in\mathbb{R}$.

$20i-21 = c^2+2cdi+(di)^2 = c^2-d^2+2cdi$

Re $-21 = c^2-d^2$ $\longrightarrow$ $-21 = \left(\frac{10}{d}\right)^2-d^2$

$20 = 2cd \longrightarrow c=\frac{10}{d}$

$d^4-21d^2-100=0. \qquad d^2 = \dfrac{+21\pm\sqrt{(-21)^2+4\cdot 100}}{2} =$

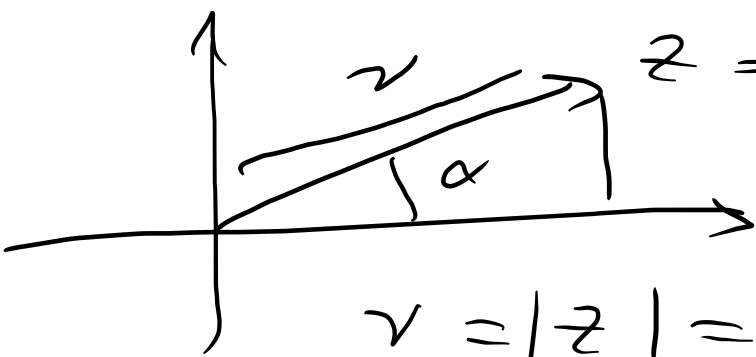
$= \dfrac{21\pm 29}{2} = \begin{matrix} 25 \\ \cancel{-4} \end{matrix}$ (d valós) $\longrightarrow d=\pm 5$

$d = 5 \Rightarrow c = \frac{10}{5} = 2$

$-5 \Rightarrow c = \frac{10}{-5} = -2$

$\pm(2+5i)$

TRIG. ALAK.



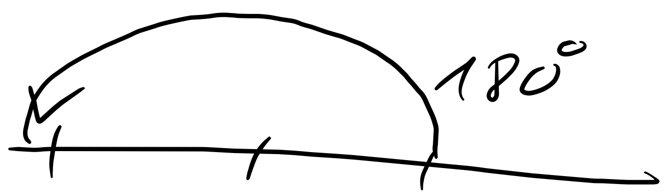
$z = a+bi = r(\cos\alpha + i\sin\alpha)$

$z \neq 0.$

$r = |z| = \sqrt{a^2+b^2}$

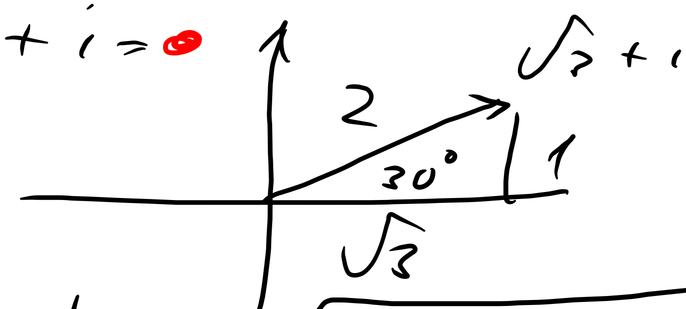
$1-i = \sqrt{2}\,(\cos 135^\circ + i\sin 135^\circ)$

$-4 = 4\,(\cos 180^\circ + i\sin 180^\circ)$

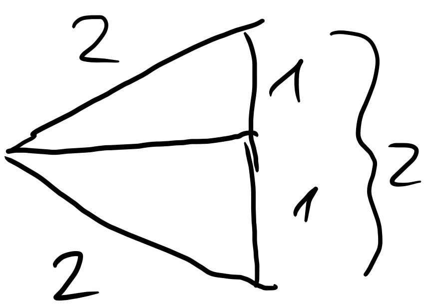


$4 = |-4|$

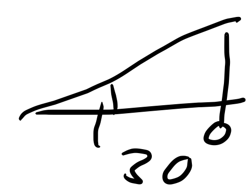
$\sqrt{3}+i = \bullet$



$|\sqrt{3}+i| = \sqrt{(\sqrt{3})^2+1^2} = \underline{\underline{2}}$

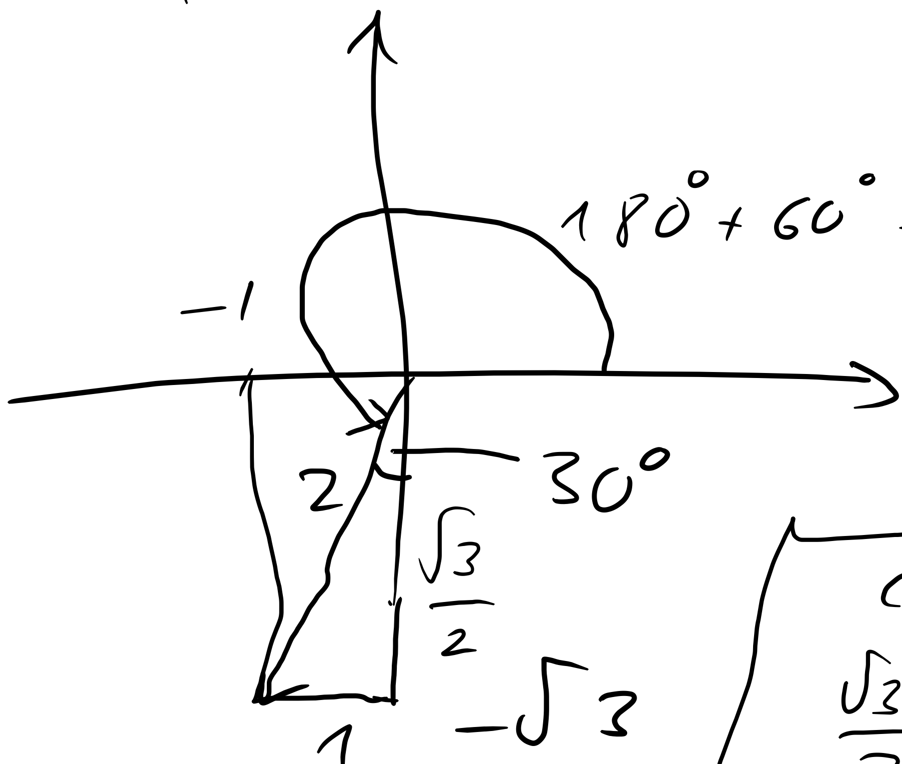


std. Δ fele $\Rightarrow$



$\bullet = 2(\cos 30^\circ + i\sin 30^\circ).$

$$|-1-\sqrt{3}\,i| = \underline{\underline{2}}$$



$$180° + 60° = \underline{\underline{240°}}$$

$$-1-\sqrt{3}\,i =$$
$$= 2(\cos 240° + i\sin 240°).$$

$$\cos(30°) - i\sin(60°) =$$

$$\frac{\sqrt{3}}{2}(1-i) = \frac{\sqrt{3}}{2}\sqrt{2}(\cos(315°) + i\sin 315°)$$

↳ [illegible]

$$\to \frac{\sqrt{6}}{2} = \frac{\sqrt{3}}{\sqrt{2}}.$$

$\sqrt{12i-5}$ vop ZH.

$$\pm(2+3i)$$