3. b17.

$$\underline{b} = (\overset{\lambda_1}{1+x},\ \overset{\lambda_2}{1+x^2},\ \overset{\lambda_3}{1+x+x^2})$$

$$[9+6x+7]_{\underline{b}} \qquad \begin{array}{c} 1 \\ x \\ x^2 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 7 \end{array}\right]$$

Egyért. megoldás 2, 3, 4.

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \qquad 2\cdot 3\cdot 4 = \underline{\underline{24}}$$

$$\begin{bmatrix} a & c & d \\ 0 & b & e \\ 0 & 0 & -a-b \end{bmatrix} \quad 5 \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad 5 \text{ db.}$$

bázis.

≤ 10. fokú, $f(1)=f(2)=f(3)=f(4)$

11 3 db F egyenlet. $11-3=8$.

$\le 10.$ fokú

1	$x^i(x-1)(x-2)(x-3)(x-4)$, $0 \le i \le 6$, csupa ... a foka	$(x-1)(x-2)(x-3)$	$(x-1)(x-2)$	x

11

8 db ?

≥ 8 $\boxed{8}$

≤ 8: $f(1)=f(2)=f(3)=f(4)$

≤ 9: $f(1)=f(2)=f(3)$

≤ 10: $f(1)=f(2)$

$\mathbb{R}$ fölött

$$\begin{bmatrix} a & b+ci \\ d+ei & f+gi \end{bmatrix}$$

$\boxed{7}$ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & i \\ 0 & 0 \end{bmatrix}$ stb.

$\le 8.$ fokú $\mathbb{R}[x]$ $\boxed{9}$

i, $i+1$, $-i+1$ gyöke

i gyöke $\Rightarrow$ $\bar{i} = -i$ is gyöke.

$$(x^2+1)\underbrace{(x-i+1)(x-i-1)}_{\in \mathbb{R}[x]}(a_0 + a_1x + \dots + a_4x^4)$$

a_i tetszőleges, függetlenül választhatók

$\boxed{5}$

$\mathbb{R}^{10}$ $\quad a_1^2 + \dots + a_6^2 = 0 \Rightarrow a_1 = \dots = a_6 = 0$

$10 - 6 = \boxed{4}$

(b) 3 $U = \{$ első 3 koord összege 0 $\} \subseteq \mathbb{R}^4$

3 $W = \{$ utolsó — " — $\}$

$U \cap W = ?$

$U + W = ?$

$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in U \cap W \quad \begin{matrix} a+b+c=0 \\ b+c+d=0 \end{matrix}$

$\begin{bmatrix} -b-c \\ b \\ c \\ -b-c \end{bmatrix}$ 2-dim (2 szabad változó)

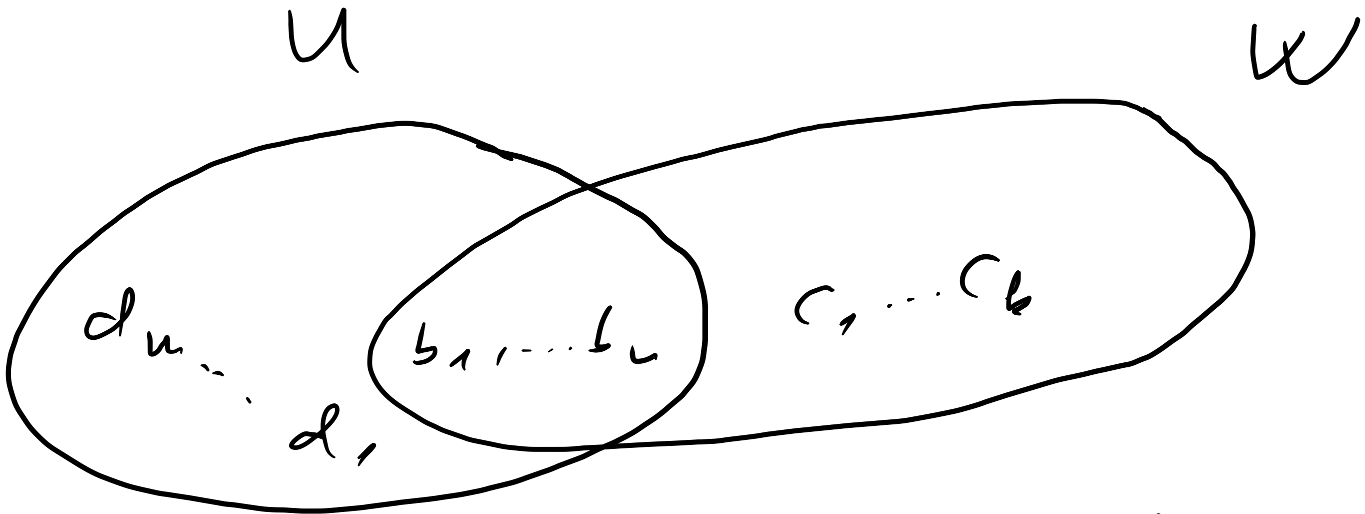
$\dim(U+W) = 3+3-2 = 4$

$\Rightarrow U+W = \mathbb{R}^4$

(nem lehet valódi altér). $\quad U \cap W = \{0\}$

(c) $U = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \middle| \ a+b+c+d=0 \right\}$ 3 $\quad W = \left\{ \begin{bmatrix} a \\ a \\ a \\ a \end{bmatrix} \middle| \ a \in \mathbb{R} \right\}$ 1

$\Rightarrow U+W = \mathbb{R}^4$. $\quad 3+1-0=4$



$(n+m)+(n+k)-n =$

$= n+m+k.$

$\dim(U+W) = \dim U + \dim W - \dim(U \cap W).$

$b_1, \ldots, b_n$ B $U \cap W$

b_1 — b_n és c_1 — c_k B W-ben

b_1 — b_n és d_1 — d_m B U-ban.

<u>Áll</u>. b_1 —, b_n, c_1 — c_k, $d_1 \ldots d_m$ B $U+W$-ben.

F, G HF.

$A : U \to W$ lin. lek. ha

-1. V és W UGYANAZ / fölött.

1. összegtartás: $A(u+w) = A(u) + A(w)$

2. λ-szoros tartás $(\forall \lambda)$ $A(\lambda u) = \lambda A(u)$.

[0. $A(0_V) = 0_W$, de ez követ már].

III/1 (a) NEM $\mathbb{R} \neq \mathbb{C}$

(b) $A(u) = 3u$

$$3(u + w) \stackrel{?}{=} 3u + 3w \quad ✓$$

$$A(\lambda u) = \lambda A(u)$$

$$3(\lambda u) \stackrel{?}{=} \lambda (3u) \quad ✓$$

Igen, ez lin.

négyzetre emelés NEM $(v+w)^2 \stackrel{?}{=} v^2 + w^2$ NEM

$$(1+1)^2 \neq 1^2 + 1^2$$

(g) egybevágósági trafó

$0 \longrightarrow 0$

eltolás : csak ha helybenhagyás.

forg : csak 0 körül

tükr., vetítés : egyenes $\ni$ origó

pontra tükr. : csak 0-ra.

$A : V \longrightarrow W$

$\underline{b} = (b_1, \dots, b_n)$ B V-ben

$\underline{c} = (c_1, \dots, c_m)$ B W-ben

$m = \dim W$ sor !!

$n = \dim V$ oszlop !!

$$[A] = \begin{array}{c|c} & A(b_1), \dots, A(b_n) \\ \hline \begin{array}{c} c_1 \\ \vdots \\ c_m \end{array} & \begin{pmatrix} \lambda_1 & & \\ \vdots & & \\ \lambda_m & & \end{pmatrix} \end{array}$$

$A(b_1) = \lambda_1 c_1 + \dots + \lambda_m c_m$.

stb.

LIN. EGY. RR. Honlapra!

$A(v) = 3v$ $\quad A: \mathbb{C}_{\mathbb{R}} \to \mathbb{C}_{\mathbb{R}}$. ható

1 bázis : a szokásos $(1, i)_{\underline{b}}$.

$$\begin{array}{c|cc} & A(1) & A(i) \\ \hline 1 & \lambda_1 & \mu_1 \\ i & \lambda_2 & \mu_2 \end{array} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = [A]_{\underline{b}/\underline{b}} = [A]_{\underline{b}}$$

$A(1) = 3 \cdot 1 = 3 = \lambda_1 \cdot 1 + \lambda_2 \cdot i$

$A(i) = 3i = \mu_1 1 + \mu_2 \cdot i$

$B(v) = (1+i)v$

$$\begin{array}{c|cc} & B(1) & B(i) \\ \hline 1 & 1 & -1 \\ i & 1 & 1 \\ \hline & 1+i & i-1 \end{array}$$

$B(1) = 1+i$

$B(i) = (1+i)i = i-1$

$$C(u) = \overline{u}$$

$$\begin{array}{c|cc} & C(1) & C(i) \\ 1 & & \\ i & & \end{array} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{array}{cc} 1 & -i \end{array}$$

$$\overline{1} = 1 = 1 + 0 \cdot i$$

$$\overline{i} = -i = 0 - 1 \cdot i$$

$$A : \mathbb{R}^4 \longrightarrow \mathbb{R} \qquad \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \longrightarrow a_1 + a_2 + a_3 + a_4$$

(e_1, e_2, e_3, e_4) standard $\qquad (1)$

$$\| \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ stb.}$$

$$\begin{array}{c|cccc} & A(e_1) & A(e_2) & A(e_3) & A(e_4) \\ 1 & 1 & 1 & 1 & 1 \end{array}$$

$$A(e_1) = 1 + 0 + 0 + 0 = 1 = 1 \cdot 1$$

stb.

Ha (2) bázis $\mathbb{R}$-ban

$$\left(\tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2} \right)$$

lesz.

$A(M) = M^T$ $\quad \mathbb{R}^{2\times 2} \longrightarrow \mathbb{R}^{2\times 2}$ $\quad$ bázi: noscíos

$\underline{b} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right)$

$\qquad\quad b_1 \qquad\quad b_2 \qquad\quad b_3 \qquad\quad b_4$

$$\begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{array} \begin{array}{c} A(b_1) \; \ldots \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$A(b_1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = b_1$

$A(b_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = b_3$

$A(b_3) = b_2$, $A(b_4) = b_4$

$\leq 3.$ fokú

$A : \mathbb{R}[x] \longrightarrow \mathbb{C}$ $\qquad$ $\mathbb{R}$ fölött

$f(x) \longrightarrow f(i)$

<u>Pl</u>. $f(x) = x^2 + 2$

$\underline{b} = (1, x, x^2, x^3)$ $\qquad$ $\underline{c} = (1, i)$

$A(f) = i^2 + 2 = \underline{\underline{1}}$

$$\begin{array}{c} \\ 1 \\ i \end{array} \begin{array}{c} A(1) \quad A(x) \quad A(x^2) \quad A(x^3) \\ \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \end{array}$$

$A(1) = 1$

$A(x) = i$

$A(x^2) = i^2 = -1$

$A(x^3) = i^3 = -i$

$A(f) = f'$ $\leq 3.$ fokú $\mathbb{R}$.

$(1, x, x^2, x^3)$ TRANSZFORMÁCIÓ!!!

$$\begin{array}{c|cccc} & A(1) & A(x) & A(x^2) & A(x^3) \\ \hline 1 & & & & \\ x & & & & \\ x^2 & & & & \\ x^3 & & & & \end{array} \qquad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

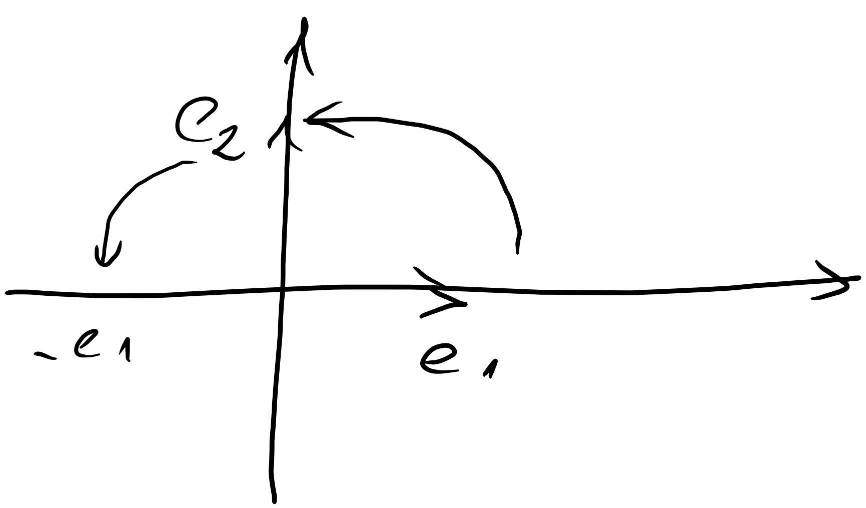
$A(1) = 1' = 0$

$A(x) = x' = 1$

$A(x^2) = (x^2)' = 2x$

$A(x^3) = (x^3)' = 3x^2$

F : $90°$ forg

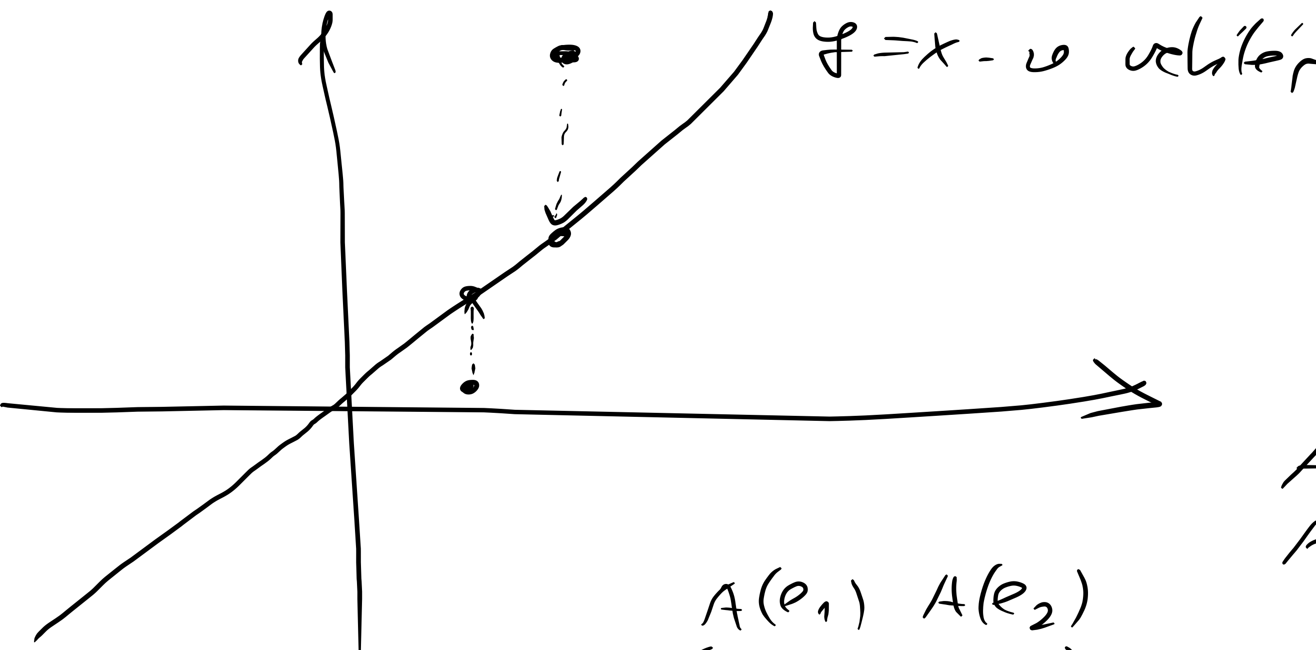


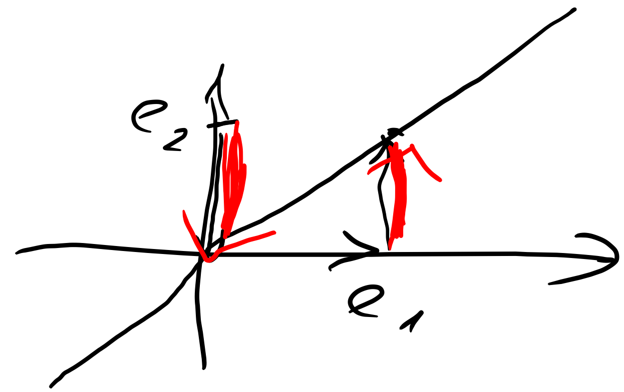
$F(e_1) = e_2$

$F(e_2) = -e_1$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

A: függőleges vetítés $y = x$-re.



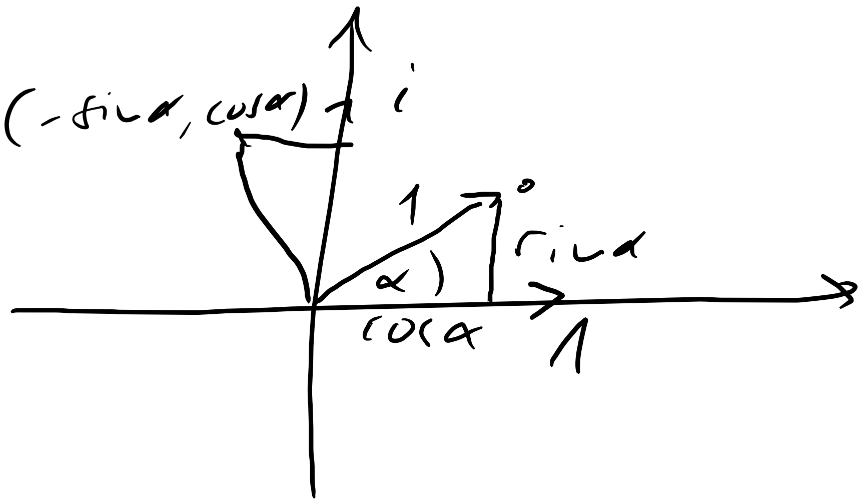


$A(e_1) = (1,1) = e_1 + e_2$

$A(e_2) = \text{origó}$

$$\begin{array}{c|cc} & A(e_1) & A(e_2) \\ \hline e_1 & 1 & 0 \\ e_2 & 1 & 0 \end{array}$$

α forg mátrixa



$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$e_1 \longrightarrow (\cos\alpha, \sin\alpha)$

Ugyanez komplex számokkal

$A(w) = w \cdot (\cos\alpha + i\sin\alpha)$

$1 \longrightarrow \cos\alpha + i\sin\alpha$

$i \longrightarrow i(\cos\alpha + i\sin\alpha) =$

$= -\sin\alpha + i\cos\alpha$

III/2.

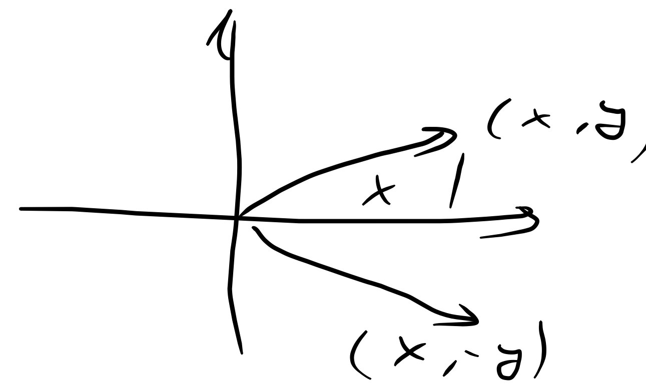
$$[A] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A = ?$$

$$[A]\begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$A : (x, y) \longmapsto (x, y)$ helybenhagyás.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

x-tengelyre tükrözés.



HF III/2.

III/3. Bázistrafó képletével.

III/8, 11.

III/7.