III/1-10.

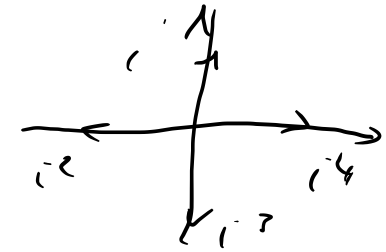
$i^{1919} = ?$ $\quad i^3 = -i$ $\qquad 1919 \equiv ?\ 3 \mod 4$?

$1 + i + i^2 + \dots + i^{1919} = ?$ Mértani sor $\qquad \dfrac{\overset{1}{i^{1920}} - 1}{i-1} = \underline{\underline{0}}$

$\varepsilon_1, \dots, \varepsilon_n$ az n-edik egységgyökök

$\varepsilon_1 + \dots + \varepsilon_n = ?$

$\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n = ?$



$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$

$i^5 = i, \quad i^6 = -1, \quad i^7 = -i, \quad i^8 = 1$

stb. 4-esével PERIODIKUS.

$\underbrace{\underbrace{1 + i + i^2 + i^3}_{1 \quad i \quad -1 \quad -i}}_{0} + \underbrace{\qquad\qquad}_{0} + \underbrace{i^{1916} + i^{1917} + i^{1918} + i^{1919}}_{0}$

n. egységgyökös $\sqrt[n]{1}$ értékei

$$\varepsilon_j = \cos j\frac{2\pi}{n} + i\sin j\frac{2\pi}{n} = \varepsilon_1^j \quad \text{E.A.}$$

$$\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_n = ? = \varepsilon_1 + \varepsilon_1^2 + \dots + \varepsilon_1^n = \varepsilon_1\left(\frac{\varepsilon_1^n - 1}{\varepsilon_1 - 1}\right)$$

$\varepsilon_1^n \overset{?}{=} 1$ $= \underline{\underline{0}}$

ha $\varepsilon_1 \neq 1$

$\varepsilon_1 - 1 \overset{?}{=} 0$

$n=1 \Rightarrow \varepsilon_1 = 1$

$n = 1 \Longrightarrow$ összeg $\boxed{1}$

$$\varepsilon_1 \varepsilon_2 \dots \varepsilon_n = ?$$

$$\| $$

$$\varepsilon_1^{1+2+\dots+n} = \varepsilon_1^{\frac{n(n+1)}{2}} = \left(\varepsilon_1^n\right)^{\frac{n+1}{2}} = 1^{\frac{n+1}{2}} = 1 \quad ??$$

$n=2$ $\varepsilon_1 = -1$ $\varepsilon_2 = 1$ $\varepsilon_1\varepsilon_2 = \underline{\underline{-1}}$

egész???

Ha $\frac{n+1}{2}$ egész, azaz n páratlan, akkor $\boxed{1}$

Ha n páros? -1 lesz, de miért?

Trig alakban $\varepsilon_j = \varepsilon_1^j$ szöge $j\frac{2\pi}{n}$

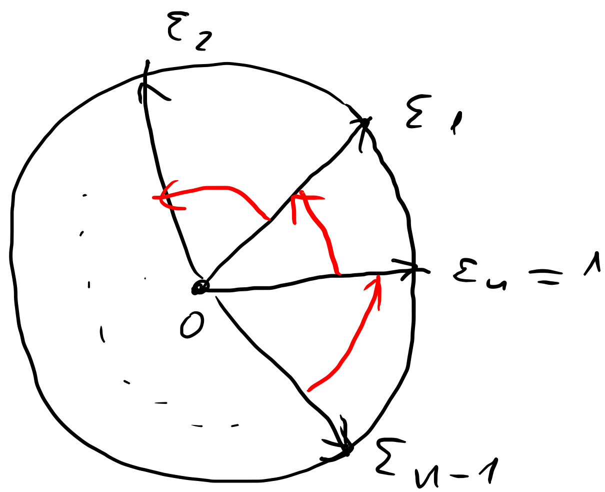
$(1+2+\dots+n)\frac{2\pi}{n}$ a

szorzat szöge $2\pi \cdot \frac{n+1}{2}$

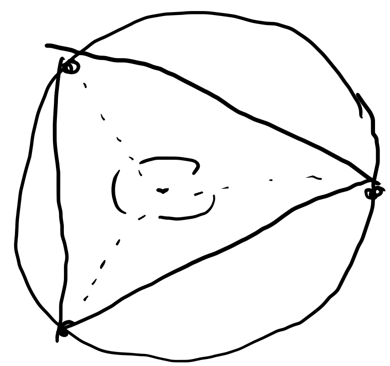
ups: $180°$! $\boxed{-1}$

$S = \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_n = ? \,/\, 0\,?$ $\quad n > 1$

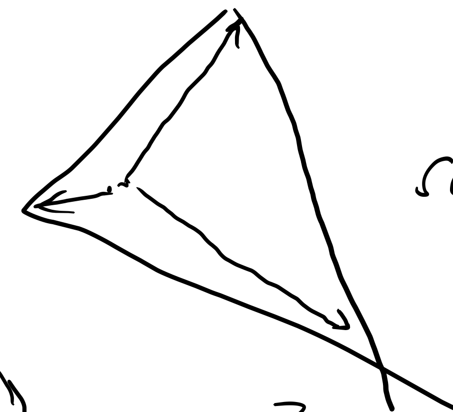
$\varepsilon_2 = \varepsilon_1^2, \quad \varepsilon_n = \varepsilon_1^n$



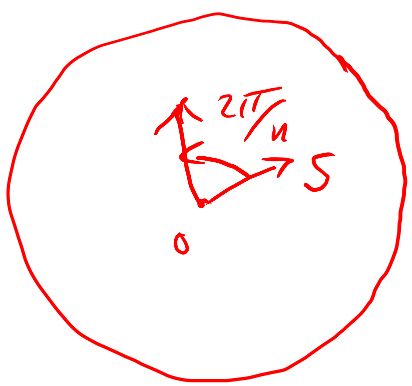
Szp-ba?



$n = 3$



súlypont!



$\frac{2\pi}{n}$ forg: összeg saját magába megy

$\Rightarrow S = 0. \quad (n > 1)$

$(\underbrace{\varepsilon_1 + \ldots + \varepsilon_n}_{= S})\,\varepsilon_1 = S$

$= \varepsilon_1^2 + \varepsilon_1^3 + \ldots + \varepsilon_1^n + \varepsilon_1^{n+1}, \quad \varepsilon_1^{n+1} = \varepsilon_1$

$S\varepsilon_1 = S$

vagy $S = 0$

vagy $\varepsilon_1 = 1 \Rightarrow n = 1.$

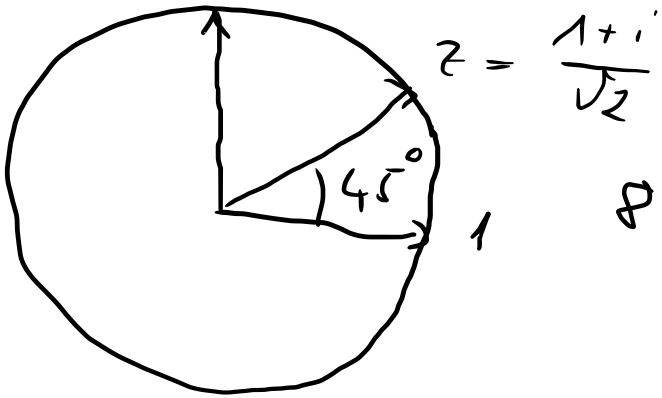
z	REND z-nek hány hatv.?	JÓ KITEVŐ $n=?\ z^n=1$	REND egészek poz. n: $z^n=1$
1	1	$\forall n$	1
-1	2	$2\mid n$	2
i	4	$4\mid n$	4
$1+i$	∞	$0\mid n$ $(n=0)$	nincs
$\frac{1+i}{\sqrt{2}}$	8	$8\mid n$	8
$\cos \sqrt{2}\cdot 360° + i\sin \sqrt{2}\cdot 360°$	∞	$0\mid n$	nincs
$\cos 336° + i\sin 336°$	15	$15\mid n$	15

$(1+i)^2=+2i$, $(1+i)^4=-4$, … $|1+i|=\sqrt{2}$ $|(1+i)^n|=\sqrt{2}^n=1 \iff n=0.$

nincs 1-el egyenlő, a hosszuk is más.

$(z \neq 0)$

$|z| \neq 1 \Longrightarrow$ $\forall z$ hatványa kül.



8-adik

$$r(\cos\alpha + i\sin\alpha) = s(\cos\beta + i\sin\beta) \Longleftrightarrow$$
$$r = s$$
$$\alpha - \beta = k\cdot 360^\circ$$

$$\left(\cos \sqrt{2}\, 360^\circ + i \sin \sqrt{2}\, 360^\circ\right)^n =$$
$$= \cos n\sqrt{2}\, 360^\circ + i \sin n\sqrt{2}\, 360^\circ \overset{?}{=} 1$$
$$1 = \cos 0^\circ + i \sin 0^\circ$$
$$\Longleftrightarrow \quad k \cdot \cancel{360^\circ} = n\sqrt{2} \cdot \cancel{360^\circ}$$
$$\sqrt{2} = \frac{k}{n} \qquad \sqrt{2} \text{ irrac.} \qquad \lightning$$

HF $\forall z$ hatványa kül.

Tehát $\boxed{n = 0}$ ($n \neq 0$)

$|z| \neq 1 \Rightarrow \forall z$ hatvány kül.

szög$/360°$ irrac $\Rightarrow$ — " —

szög$/360° = p/q$ $(p,q)=1$ $\Rightarrow$ REND $= \boxed{q}$

$(\cos 336° + i\sin 336°)^n = 1$ $\qquad$ poz. $\qquad$ 15

$\frac{336°}{360°}$ $\qquad n \cdot \cancel{336°}_{14} = k \cdot \cancel{360°}_{15}$ $\qquad$ Melyik a legkisebb n, amikor van ilyen k?

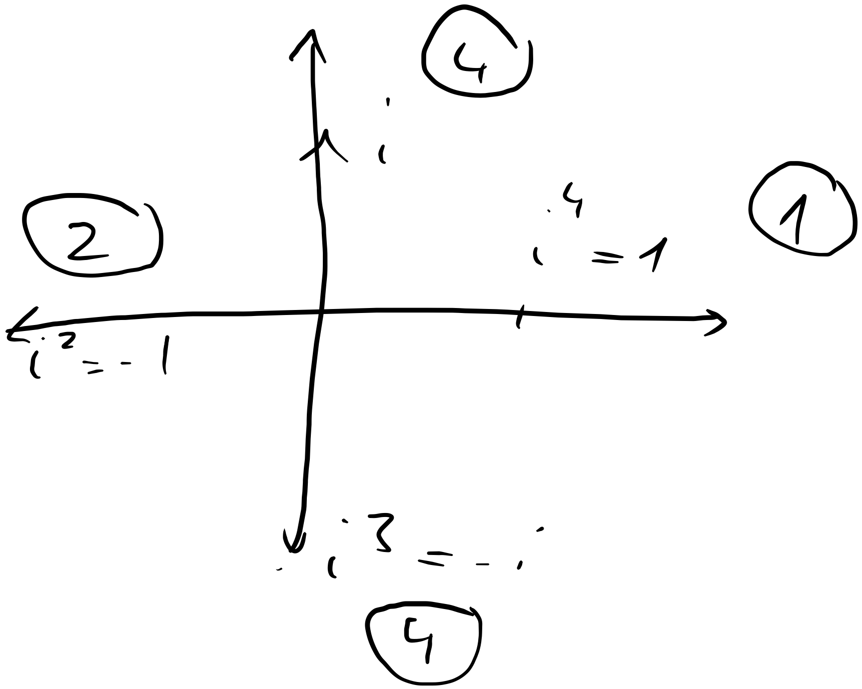
$14n = 15k \Leftrightarrow \left.\begin{array}{l} 15 \mid 14n \\ (15,14)=1 \end{array}\right\} \Rightarrow 15 \mid n$

$\left.\begin{array}{l} a \mid bc \\ a \nmid b \end{array}\right\} \not\Rightarrow a \mid c$

$\left.\begin{array}{l} a \mid bc \\ (a,c)=1 \end{array}\right\} \Rightarrow a \mid c$

$\left.\begin{array}{l} 6 \mid 2\cdot 3 \\ 6 \nmid 2 \end{array}\right\} \not\Rightarrow 6 \mid 3$

ε PRIMITÍV n-edik egységgyök $\Leftrightarrow$ rendje n



$i, -i$ PRIMITÍV 4. egységgyökök

-1 az eredeti primitív 2. e.gy.

$1, i, -1, -i$ összes 4. egységgyök

$\varphi(4) = 2$ 2db primitív 4. e.gy.

↑ Euler-fv.

$\cos j\frac{2\pi}{n} + i\sin j\frac{2\pi}{n}$ primitív n. e.gy.

$\iff (j, n) = 1$.

ε pr. 512. $\qquad o(-i\varepsilon) \overset{?}{=} \boxed{512}$

$\Updownarrow$

$o(\varepsilon) = 512$

$\begin{cases} \varepsilon \text{ hossza} = 1 \\ \text{szöge}/360^\circ = \dfrac{k}{512} \end{cases}$ $\qquad (k, 512) = 1 \Longleftrightarrow k$ páratlan

$512 = 2^9$

$k - 128$ páratlan

$$\varepsilon = \cos\frac{k\,360^\circ}{512} + i\sin\frac{k\cdot 360^\circ}{512}$$

$$-i = \cos(-90^\circ) + i\sin(-90^\circ)$$

$$-i\varepsilon = \cos\Big(\underbrace{\frac{k\cdot 360^\circ}{512} - 90^\circ}\Big) + i\sin(\quad)$$

$$= \frac{k\cdot 360^\circ - 90^\circ\cdot 512}{512} = 360^\circ\left(\frac{k - \frac{1}{4}\cdot 512}{512}\right) =$$

$$= \frac{k - 128}{\boxed{512}}\cdot 360^\circ \Longleftarrow (k-128, 512) = 1$$

$o(\varepsilon) = 512$

$o(-i\varepsilon) = ?$

II. mo. $\quad (-i\varepsilon)^{?} = 1$

$$(-i\varepsilon)^{512} = \underbrace{(-i)^{512}}_{\substack{\| \\ (-i)^{4\cdot 128} \\ \| \\ 1}} \underbrace{\varepsilon^{512}}_{\substack{\| \\ 1}} = \underline{\underline{1}}$$

512 ~~jó~~ rend kitevője $-i\varepsilon$-nak

$o(-i\varepsilon) \mid 512 = 2^9 \Rightarrow o(-i\varepsilon) = 2^k \quad k \le 9$

HA NEM 512 $\Rightarrow k < 9 \Rightarrow o(-i\varepsilon) \mid 2^8$

$\Rightarrow \underbrace{(-i\varepsilon)^{256}}_{\substack{\| \\ (-i)^{256} = 1}} = 1 \quad \Big| \Rightarrow \varepsilon^{256} = 1 \quad ↯$

$o(\varepsilon) = 512.$