$a+bi$ olyan, mint ha i ismeretlen lenne,
de $i^2=-1$.

$z=a+bi \qquad \bar z = a-bi$ konjugált

$|z| = \sqrt{a^2+b^2}$ abszolútérték.

$(1+i)(3-2i)$

$$\frac{1}{a+bi} = \frac{a-bi}{(a+bi)(a-bi)} = \frac{a-bi}{a^2+b^2} \leftarrow a^2-(bi)^2$$

$$\frac{1}{z} = \frac{\bar z}{|z|^2}$$

i eltűnjön a nevezőből: $a-bi$-vel bővítjük a törtet.

$$\frac{1}{i} = \frac{1\cdot(-i)}{i(-i)} = \underline{\underline{-i}}$$

$$\frac{1+i}{3-2i} = \frac{(1+i)(3+2i)}{(3-2i)(3+2i)} = \frac{3+2i+3i-2}{3^2+2^2} = \underline{\underline{\frac{1}{13}+\frac{5}{13}i}}$$

$$\left|\frac{\overline{(4+i)}}{4+i}\right| = \left|\frac{4-i}{4+i}\right| = \left|\frac{(4-i)^2}{4^2+1^2}\right| = \left|\frac{16-8i+i^2}{17}\right| = \left|\frac{15}{17}-\frac{8}{17}i\right| =$$

$$\left|\frac{(1+1526i)^{100}}{(1-1526i)^{100}}\right| = ?$$

$(1+i)(3-2i) = 3-2i+3i-2i^2 =$
$= 3+i+2 = \underline{\underline{5+i}}$

$$= \sqrt{\left(\frac{15}{17}\right)^2+\left(\frac{-8}{17}\right)^2} = \underline{\underline{1}}$$

$z = a + bi \qquad \bar{z} = a - bi \qquad |z|^2 = a^2 + b^2$ ← örrzegtartó!

$z \cdot \bar{z} = |z|^2$ (1) $\qquad \bar{\bar{z}} = z$ (2) $\qquad \overline{z + w} = \bar{z} + \bar{w}$ (3)

$\overline{zw} = \bar{z} \cdot \bar{w}$ (4) $\qquad |zw| = |z||w|$ (5) $\qquad z = \bar{z} \iff z \in \mathbb{R}$ (6)

$|z| = |\bar{z}|$ (7) $\quad \left(|a+bi| = |a-bi| = \sqrt{a^2+b^2} \right) \qquad |z/w| = |z|/|w|$ (8)
$w \neq 0$

$$\left| \frac{\overline{4+i}}{4+i} \right| \overset{(8)}{=} \frac{|\overline{4+i}|}{|4+i|} \overset{(7)}{=} \frac{|4+i|}{\underbrace{|4+i|}_{\neq 0}} = \underline{\underline{1}}$$

$|z^2| = |z \cdot z| = |z||z| = |z|^2$

$|z^3| = |z|^3 \quad (z \cdot z^2)$

$|z^4| = |z|^4$ (9)

$$\left| \frac{(1+1526i)^{100}}{(1-1526i)^{100}} \right| \overset{(8)}{=} \frac{|(1+1526i)^{100}|}{|(1-1526i)^{100}|} \overset{(9)}{=} \frac{|(1+1526i)|^{100}}{|(1-1526i)|^{100}} =$$

$$= \left(\frac{|1+1526i|}{|\underbrace{1-1526i}_{\overline{1+1526i}}|} \right)^{100} \overset{(7)}{=} \left(\frac{|1+1526i|}{|1+1526i|} \right)^{100} = \underline{\underline{1}}.$$

$$(1+i)^2 = 1^2+2i+i^2 = \underline{\underline{2i}}$$

$$(1+i)^4 = (2i)^2 = \underline{\underline{-4}}$$

$$(1+i)^6 = (1+i)^4(1+i)^2 = -4\cdot 2i = -8i$$

$$(1+i)^8 = ((1+i)^4)^2 = (-4)^2 = 16$$

$$(1+i)^{12} = (1+i)^8(1+i)^4 = 16\cdot(-4) = -64$$

$$(1+i)^{1241} = (1+i)^{1240}(1+i) = (-4)^{310}(1+i) = \underline{\underline{2^{620}+2^{620}i}}$$

$$\rightarrow (1+i)^{4k} = (-4)^k$$

2\. fokú $\Rightarrow$ ≤ 2 gyök!

$\leftarrow$ gyökt. alak!

$x^2+1=0 \rightsquigarrow x = \underline{\underline{i}}$ jó, $x = -\underline{\underline{i}}$ is jó.

$x^2+1 = (x+i)(x-i)$

$\sqrt{-1} = \pm i$

$x^2 = -4$
$x = \pm 2i$

$x^2 = -12 \rightarrow \underline{\underline{x = \pm\sqrt{12}\, i}}$

$x^2+2x+2=0$
$x^2+2ix-1=0$ } megoldóképlet.

$r<0 \quad r\in\mathbb{R}$
$\sqrt{r} = \pm\sqrt{-r}\, i$

Negatív valósból tudunk $\sqrt{\ }$-öt vonni.

$$1\cdot x^2+2x+2=0 \qquad x=\frac{-2\pm\sqrt{4-4\cdot 2}}{2}=\frac{-2\pm\sqrt{-4}}{2}=$$

$$=\frac{-2\pm 2i}{2}=\underline{\underline{-1\pm i}}$$

$$1\underbrace{\big(x-\underset{\text{gyök}}{(-1+i)}\big)\big(x-\underset{\text{gyök}}{(-1-i)}\big)}_{\text{gyöktényezős alak}}=(\underbrace{x+1}_{a}-\underset{b}{i})(\underbrace{x+1}_{a}+\underset{b}{i})=$$

EO:

$$=(x+1)^2-i^2=x^2+2x+1+1$$

$$x^2+2ix-1=0 \qquad x=\frac{-2i\pm\sqrt{(2i)^2-4(-1)}}{2}=\frac{-2i}{2}=\underline{\underline{-i}}$$

kétszeres gyök.

$$\|$$

$$(x+i)^2$$

gyökt. alak.

Meg tudunk oldani valós együtthatós 2. fokút.
Mert a legrosszabb esetben negatív valós $\sqrt{\ }$-a kell.

$$\sqrt{20i-21} = ? \qquad c+di = \underline{\underline{\pm(2+5i)}} \checkmark$$

$$(c+di)^2 = 20i-21 \qquad \underline{c, d \in \mathbb{R}} \text{ ismeretlen}$$

$$c^2+2cdi+(di)^2 = c^2-d^2+2cdi$$

Algebrai négyzetgyökvonás $\mathbb{C}$-ben

Mikor egyenlő két komplex szám?

$$\left.\begin{array}{ll} \mathrm{Re}: & c^2-d^2 = -21 \\ \mathrm{Im}: & 2cd = 20 \end{array}\right\}$$

i együtthatója !!

$$c = \frac{10}{d}$$

$$\left(\frac{10}{d}\right)^2 - d^2 = -21 \qquad d^2\text{-re 2. fokú egy.}$$

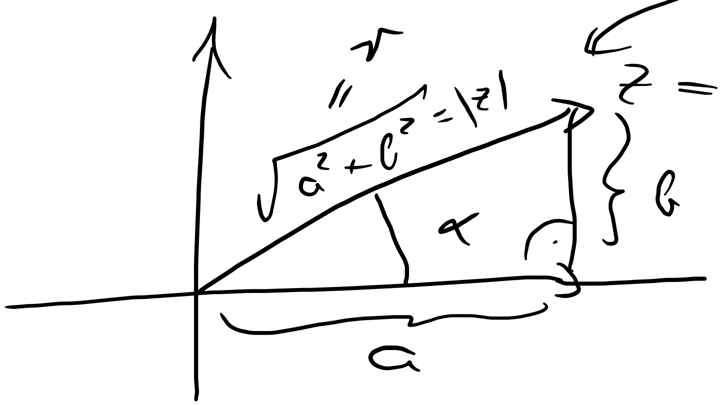
$$d^4 - 21d^2 - 100 = 0 \qquad d^2 = \frac{21 \pm \sqrt{21^2+400}}{2} = \frac{21 \pm \sqrt{841}}{2}$$

$$= \frac{21 \pm 29}{2} = \begin{cases} 50/2 = 25 & d = \pm 5 \Rightarrow c = \frac{10}{d} = \pm 2 \\ -8/2 = -4 & \leftarrow d \text{ valós szám.} \end{cases}$$

$\underline{\text{Ell}}$: $(2+5i)^2 = 4+20i-25$ ✓

HF: $x^2+(i-2)x+(6-6i)=0$.
megoldóképlettel.

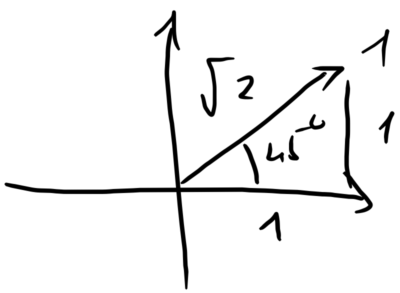
Trig. alak.



(a,b)

$z=a+bi$

$z=r(\cos\alpha+i\sin\alpha)$ $\quad r=|z|!$



$1+i=\sqrt{2}(\cos 45°+i\sin 45°)$

$1-i$
$\sqrt{3}+i$
$-1-\sqrt{3}i$

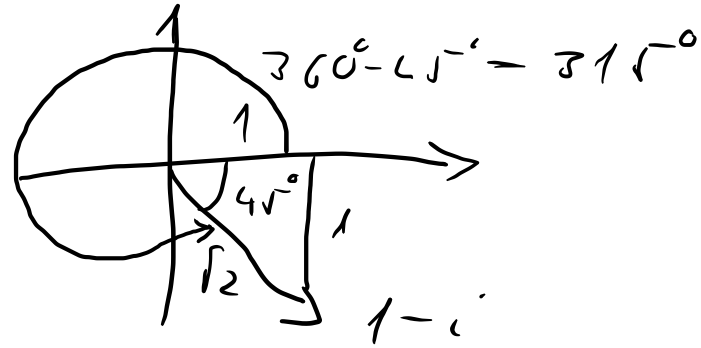
Trig. alakra hozás

1. RAJZOLJUK LE!
2. $|z|=\sqrt{a^2+b^2}$ $\to$ megvan r
3. szög: dönti geo vagy arc tg.

$1-i$

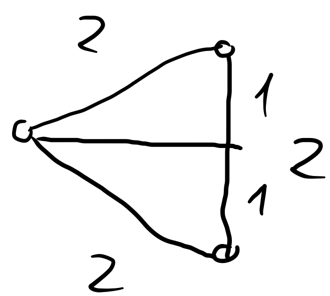
$|1-i| = \sqrt{2}$

$1-i = \sqrt{2}(\cos 315° + i \sin 315°)$

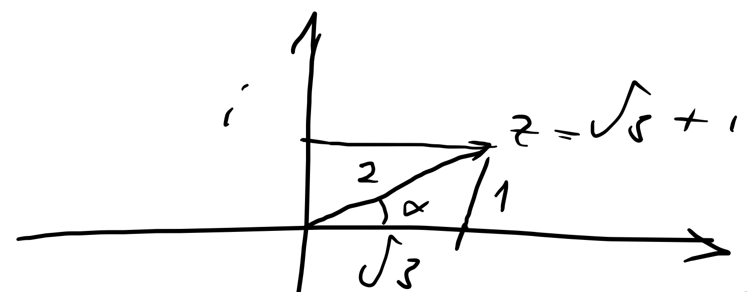


$(= \sqrt{2}(\cos(-45°) + i \sin(-45°)))$ ekvivalentní [illegible]

$\sqrt{3} + i$



$\| \quad 2(\cos 30° + i \sin 70°)$



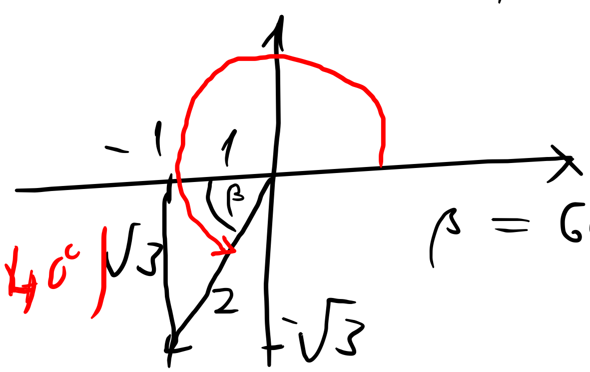
$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = 2$

$\alpha = ? \quad \alpha = 30°$

$-1 - \sqrt{3}\, i$

$|-1-\sqrt{3}\, i| = 2$

$-1 - \sqrt{3}i = 2(\cos 240° + i \sin 240°)$



$\beta = 60°$

+-nak kell.

$$\cos 30^\circ - i \sin 60^\circ = \frac{\sqrt{3}}{2} - i\,\frac{\sqrt{3}}{2} =$$

2 szög nem egyenlő

$$= \frac{\sqrt{3}}{2}\left(1 - i\right) = \frac{\sqrt{6}}{2}\left(\qquad\qquad\right)$$

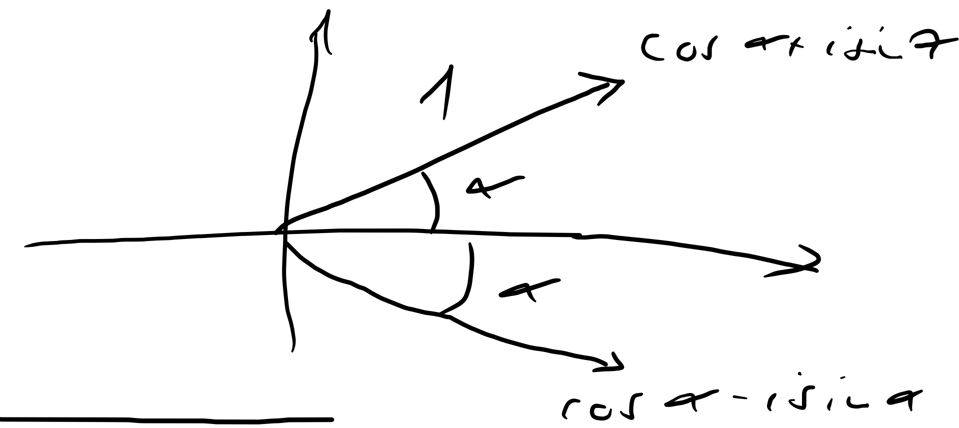
$$1 - i = \sqrt{2}\left(\cos(-45^\circ) + i \sin(-45^\circ)\right)$$

$$\cos\alpha - i\sin\alpha =$$

hossza 1 $\quad \cos^2\alpha + \sin^2\alpha = 1$

szöge: $\; -\alpha$

$$= \cos(-\alpha) + i\sin(-\alpha)$$



$$\sin\alpha + i\cos\alpha =$$

hossza 1

$$= \cos(90^\circ - \alpha) + i\sin(90^\circ - \alpha)$$